

EXAM 1 - MATH 251

Friday, September 11

YOUR NAME: _____

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Read each problem **very carefully** before starting to solve it. Each problem is worth 10 points. It is necessary to show **all** your work. Correct answers without explanations are worth 0 points. GOOD LUCK!!

1. Consider the points $P(2, 1, 5)$ and $Q(3, 4, -7)$.

(a) Find a vector $\mathbf{v}$ in the direction of $\overrightarrow{PQ}$ having length 3.

(b) Let $R(\alpha, 1, 3)$ be a third point, where α is some real number. Determine α so that the vector $\overrightarrow{PR}$ is perpendicular to $\overrightarrow{PQ}$.

2. (a) Find the angle between the two vectors $\mathbf{a} = \mathbf{i} - 2\mathbf{j} + \mathbf{k}$ and $\mathbf{b} = \mathbf{i} + \mathbf{j} - 2\mathbf{k}$ in degrees (2 decimals).

- (b) Determine the vector $\text{proj}_{\overrightarrow{OP}} \overrightarrow{OQ}$, where O is the origin, $P(3, 7, -2)$ and $Q(1, 1, -3)$.

3. (a) Find the area of the parallelogram with adjacent sides $\overrightarrow{AB}$, $\overrightarrow{AC}$, where $A(3, -1, 2)$, $B(2, 1, 5)$ and $C(1, -2, -2)$.

- (b) Find the volume of the parallelepiped with adjacent sides $\mathbf{u} = \langle -3, 5, -1 \rangle$, $\mathbf{v} = \langle 0, 2, -2 \rangle$ and $\mathbf{w} = \langle 3, 1, 1 \rangle$.

4. (a) Find an equation for the line ℓ passing through the points $P(1, 1, 1)$ and $Q(1, -1, 2)$.

(b) Find the distance of the point $M(1, 2, -3)$ from the line ℓ of Part (a).

5. (a) Write an equation for the plane that passes through the points $P(1, 1, 3)$, $Q(-1, 2, 1)$ and $R(-1, -1, -1)$.

- (b) Find the distance of the point $M(1, 3, 0)$ from the plane of Part (a).