

YOUR NAME: \_\_\_\_\_

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Read each problem **very carefully** before starting to solve it. Each problem is worth 5 points. It is necessary to show **all** your work. Correct answers without explanations are worth 0 points. GOOD LUCK!!

1. Consider the vector space  $V = \mathbb{R}^2$ . Let  $\mathbf{b}_1 = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$ ,  $\mathbf{b}_2 = \begin{bmatrix} 5 \\ -6 \end{bmatrix}$ . If  $\mathbf{x} = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$ , find  $[\mathbf{x}]_{\mathcal{B}}$ .

2. Let  $V = \mathbb{P}_2$  and consider  $\mathcal{B} = \{1 - t^2, t - t^2, 2 - 2t + t^2\}$ .

(a) Show that the vectors in  $\mathcal{B}$  are linearly independent.

(b) Find  $[\mathbf{p}(t)]_{\mathcal{B}}$  if  $\mathbf{p}(t) = 3 + t - 6t^2$ .

(c) Find the change of coordinate matrix  $P_{\mathfrak{B}}$  from  $\mathfrak{B}$  to the standard basis and compute  $P_{\mathfrak{B}}^{-1}$ .

3. Consider  $A = \begin{bmatrix} 1 & -3 & -8 & -3 \\ -2 & 4 & 6 & 0 \\ 0 & 1 & 5 & 7 \end{bmatrix}$ .

(a) Find a basis and the dimension of  $\text{Nul}A$ .

(b) Find a basis and the dimension of  $\text{Col}A$ .