

# EXAM 6 - MATH 140

DATE: Friday, December 1

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Read each problem very carefully before starting to solve it. Each question is worth 5 points. It is necessary to show your work. Correct answers without explanations are worth 0 points.

GOOD LUCK!!

1. (a) Find the exact value of the expression  $\sin(\cos^{-1}(-\frac{1}{2}))$ . (1.5 point)  
(b) Find the exact value of the expression  $\sin^{-1}(\cos(\frac{3\pi}{4}))$ . (1.5 points)  
(c) Find the exact value of the expression  $\sin(\tan^{-1}3)$ . (2 points)
2. Establish the following identities:
  - (a)  $\frac{\cos^2 \theta - \sin^2 \theta}{1 - \tan^2 \theta} = \cos^2 \theta$  (1 point)
  - (b)  $\tan \theta + \frac{\cos \theta}{1 + \sin \theta} = \sec \theta$  (2 points)
  - (c)  $\frac{\cos \theta}{1 - \tan \theta} + \frac{\sin \theta}{1 - \cot \theta} = \sin \theta + \cos \theta$  (2 points)
3. Suppose that  $\alpha = \sin^{-1}(-\frac{3}{5})$  and  $\beta = \tan^{-1}(\frac{7}{3})$ .
  - (a) Compute  $\sin \alpha$  and  $\cos \alpha$ . (1.5 points)
  - (b) Compute  $\sin \beta$  and  $\cos \beta$ . (1.5 points)
  - (c) Compute  $\sin(\alpha - \beta)$ . (1 point)
  - (d) Compute  $\cos^2(\frac{\alpha}{2})$ . (1 point)
4. Solve the following trigonometric equations in the interval  $0 \leq \theta < 2\pi$ :
  - (a)  $\sin^2 \theta = 6(\cos \theta + 1)$  (2 points)
  - (b)  $\tan(2\theta) + 2 \cos \theta = 0$  (3 points)
5. Solve each equation in the interval  $0 \leq \theta < 2\pi$ .
  - (a)  $\sec \theta = \tan \theta + \cot \theta$ . (2 points)
  - (b)  $\sin \theta - \sqrt{3} \cos \theta = 1$  (3 points)