HOMEWORK 3: SOLUTIONS - MATH 110
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Problem 1 *What is the smallest natural number n, greater than 1 , for which* $(1 \times 2 \times 3 \times \ldots \times n) + 1$ *is* not *prime?*

Solution:
We have

$$1 + 1 = 2$$
$$1 \times 2 + 1 = 3$$
$$1 \times 2 \times 3 + 1 = 7$$
$$1 \times 2 \times 3 \times 4 + 1 = 25$$

Therefore 4 is the smallest number such that $1 \times 2 \times \ldots \times n + 1$ is not a prime. ■

Problem 2 *Is it possible for an extremely large prime to be expressed as a large integer raised to a very large power? Explain.*

Solution:
Suppose an extremely large prime p could be written as a large integer k raised to a very large power n, i.e., that $p = k^n$. Since p is prime and k is a big factor of p, we must have $k = p$. But then $n = 1$, which would not be a very large power. Therefore it is not possible to write p in the form k^n with both k and n very large. ■

Problem 3 *Are there infinitely many natural numbers that are not prime? If so, prove it.*

Solution:
The set $\{4, 6, 8, 10, 12, \ldots\}$ containing all even numbers greater than 2 has infinitely many elements and its members are all non-primes. ■

Problem 4 *Today is Saturday. What day of the week will it be in 3,724 days? What day of the week will it be in 365 days?*

Solution:
Since $3,724 = 532 \times 7 + 0$, 532 full weeks will have passed and we'll be at the same exact day of the week, i.e., we'll be on a Saturday.

Since $365 = 52 \times 7 + 1$, 52 full weeks will have passed plus 1 day, so our day will be a Sunday. ■

Problem 5 *Look up your bank code on your check. Verify that it is a valid bank code.*

Solution:
My bank code is 312080941. Multiply the digits by 7,3,9,7,3,9,7,3,9, respectively, and add modulo 10. We have

$$3 \times 7 + 1 \times 3 + 2 \times 9 + 0 \times 7 + 8 \times 3 + 0 \times 9 + 9 \times 7 + 4 \times 3 + 1 \times 9 =$$

$$\begin{aligned} &= 21 + 3 + 18 + 24 + 63 + 12 + 9 \\ &\equiv 1 + 3 + 8 + 4 + 3 + 2 + 9 \\ &= 30 \\ &\equiv 0 \end{aligned}$$

■

Problem 6 *Compute* $5^{600} \pmod 7$.

Solution:
We have

$$\begin{aligned} 5^{600} &= (5^6)^{100} \\ &\equiv 1^{100} \\ &= 1. \end{aligned}$$

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