F-$\mathcal{ALCI}$: A Fully Contextualized, Federated Logic for the Semantic Web

George Voutsadakis[1,2], Jie Bao[3], Giora Slutzki[1], and Vasant Honavar[1]

[1] Department of Computer Science, Iowa State University, Ames, IA 50011
[2] School of Mathematics and Computer Science,
Lake Superior State University, Sault Ste. Marie, MI 49783
[3] Department of Computer Science,
Rensselaer Polytechnic Institute, Troy, NY 12180

Abstract. In this paper, a new version F-$\mathcal{ALCI}$ of the package-based counterpart $\mathcal{ALCIP}^-$ of the description logic $\mathcal{ALCI}$ is introduced. It allows contextualization of all the logical connectives rather than just logical negation. Moreover, a new semantics is introduced that is based on image domain relations, but is not ridden with overly restrictive conditions from the outset. One may impose additional conditions if stronger properties are required in a specific framework. These features allow more flexibility and generality in the modeling process. To show that this contextualized federated description logic is decidable, a sound and complete reduction to the description logic $\mathcal{ALCI}$ is provided.

1 Introduction

Recent efforts aimed at enriching the world-wide web with machine interpretable content and interoperable resources and services, are transforming the web into the *semantic web* [7]. The semantic web, much like the world-wide web, relies on the *network effect*, that is on leveraging the work of independent actors who contribute resources that are interlinked to form a web of resources. In short, *web pages* : *web* :: *ontologies* : *semantic web*. The ontologies that provide a basis for establishing the intended semantics of resources (databases, knowledge bases, services) that constitute the semantic web are typically developed independently to serve the needs of specific communities. They typically cover different, partially overlapping, domains of discourse (e.g., biology, medicine, pharmacology). Inevitably, the axioms that make up the ontologies are applicable within the *contexts* that are implicitly assumed by their authors. However, many application scenarios require selective use of knowledge from multiple independently developed ontology modules. For example, a group that is focused on translating discoveries that link genetic and environmental factors to specific diseases into effective therapies might need to selectively reuse the contents of an ontology created for use in one context (e.g., genetic studies) in a different, but related context (e.g., drug design). Reaping the benefits of the network effect in such a setting requires theoretically well-founded yet practically useful approaches to selective, context-sensitive reuse of knowledge from autonomous, distributed, ontology modules.

Early recognition of the importance of careful treatment of context in artificial intelligence systems [26] was followed by work on non-montonic reasoning [26, 25, 27, 29], propositional and quantificational (first order) logics of context [14, 12, 11, 13] and context-based logics for distributed knowledge representation and reasoning [19, 16, 15, 10]. More closely related to contextualizing information in the semantic web are the references [23, 8, 31, 32, 6].

More recently, several modular ontology formalisms that support, with varying degrees of success, reuse of knowledge from multiple distributed ontology modules, have been explored. Examples include distributed description logics (DDL)[9, 17], $\mathcal{E}$-Connections [22], semantic importing [28], semantic binding [34], and package-based description logics (P-DL) [2, 5]. Such frameworks typically assume that the individual ontology modules are expressed in some decidable family of description logics (DL) and provide constructs for the sharing of knowledge across ontology modules. An alternative approach to knowledge reuse relies on a particular notion of modularity of knowledge bases based on the notion of *conservative extensions* [18, 21, 20], which allows ontology modules to be interpreted using standard semantics by requiring that they share the same interpretation domain.

Existing modular ontology formalisms offer only limited ways to connect ontology modules and, hence, limited ability to reuse knowledge across modules. For instance, DDL does not allow concept construction using foreign roles or concepts, or guarantee the transitivity of inter-module concept subsumptions, known as *bridge rules*. It has recently been shown that allowing negated roles or cardinality restrictions in bridge rules or inverse bridge rules, where the bridge rules are used to connect $\mathcal{ALC}$-ontologies, makes the resulting DDL ontology undecidable [4]. $\mathcal{E}$-Connections does not allow concept subsumptions across ontology modules or the use of foreign roles. Conservative extensions [21, 20, 24] require a single global interpretation domain. Hence, the designers of different ontology modules have to anticipate all possible contexts in which knowledge from a specific module might be reused thereby precluding flexible and selective reuse of knowledge across ontology modules.

P-DL offers a richer syntax than the previous approaches but, to preserve contextuality of knowledge and transitivity of role inclusions across ontology modules, and to guarantee decidability of the resulting logic, the P-DL $\mathcal{SHOIQP}$ [5] imposes several restrictions. In particular, P-DL requires partial isomorphisms between various local domains that are related via domain relations (that is, functions that link individuals across the different local domains). Hence, it is interesting to explore whether some of these conditions can be relaxed, simplifying in the process the semantics as well as the design of federated reasoners for the resulting logic.

Against this background, this paper explores a family of logics, which we call *contextualized federated description logics* (CFDLs), with special attention to characterization of the tradeoffs between specific restrictions on semantics and some of the desirable features that are offered by P-DLs. Specifically, we focus on the language F-$\mathcal{ALCI}$, which is the contextualized federated counterpart of the P-DL $\mathcal{ALCIP}^-$, in which each of the individual ontology modules is expressed in the DL $\mathcal{ALCI}$. Some features of CFDL F-$\mathcal{ALCI}$ include:

- Provision for use of a relatively rich DL within each ontology module.
- Contextualized interpretation of each logical connective used within the DL modules (unlike in P-DLs [2, 5] where only negation is contextualized). Locality of axioms in ontology modules is obtained "for free" by its *contextualized semantics*. Thus, as in the case of P-DLs, inferences are always drawn *from the point of view* of a *witness* module. It follows that different modules might infer different consequences, based on the knowledge that they import from other modules.
- Guarantee that the results of reasoning are always the same as those obtained by a standard reasoner over an integrated ontology resulting from combining the relevant parts of the individual ontologies in a context-specific manner.
- Relaxation of the severe restrictions imposed on the P-DL semantics as much as possible while, at the same time, retaining the desirable properties of P-DLs.

In particular, we show that in the general case, when only the most relaxed restrictions are imposed on the semantics of F-$\mathcal{ALCI}$, many of the properties that one might want to satisfy, like, e.g., monotonicity of inference and preservation of unsatisfiability, are lost. Regaining these properties requires strengthening the conditions on the semantics of F-$\mathcal{ALCI}$. Thus, the major contribution of this paper is a characterization of the tradeoffs between restrictions on semantics and some of the desirable features of P-DLs. Specifically, we show that it is possible to preserve many of the desirable properties of P-DLs, while at the same time imposing milder restrictions than those used in P-DLs.

2 The Federated Description Logic Language F-$\mathcal{ALCI}$

In [5], given an ordinary description logic $\mathcal{L}$, the notation $\mathcal{LP}$ is introduced to denote its package-based counterpart, i.e., the package-based description logic which uses $\mathcal{L}$ as the logical language in each of its packages. Furthermore, the notation $\mathcal{LP}^-$ signifies that the importing of concept names and role names across packages is acyclic. In the present work, we use the prefix "F-", standing for **F**ederated, to denote a contextualized federated language and, since our discussion is limited to acyclic importing, omit the use of a superscript "$^-$" from the notation.

In this section, the syntax and the semantics of the language F-$\mathcal{ALCI}$ will be described in some detail.

2.1 The Syntax

Suppose a directed acyclic graph $G = \langle V, E\rangle$, with $V = \{1, 2, \ldots, n\}$, is given. The intuition is that its n nodes correspond to local modules of a modular ontology and its edges correspond to the importing relations between these modules. For technical reasons, we add a loop on each vertex of G.

For every node $i \in V$, the signature of the i-language always includes a set $\mathcal{C}_i$ of i-**concept names** and a set $\mathcal{R}_i$ of i-**role names**. We assume that all sets of names are pairwise disjoint. Out of these, a set of *i-concept expressions* $\widehat{\mathcal{C}}_i$ and a set of *i-role expressions* $\widehat{\mathcal{R}}_i$ are built.

Recall that the description logic $\mathcal{ALCI}$ allows concept expressions that are constructed recursively from its signature symbols, i.e., its role and concept names, using negation, conjunction, disjunction, value and existential restriction and inverses of role names. Its formulas are subsumptions between concept expressions.

The syntax of the description logic F-$\mathcal{ALCI}$ is defined as follows:

Definition 1 (Roles and Concepts) *The set of i-**roles** or i-**role expressions** $\widehat{\mathcal{R}}_i$ consists of expressions of the form R, R^-, with $R \in \mathcal{R}_j, (j,i) \in E$.*

*The set of i-**concepts** or i-**concept expressions** $\widehat{\mathcal{C}}_i$, on the other hand, is defined recursively as follows:*

$$A \in \mathcal{C}_j, \top_j, \bot_j, \neg_j C, C \sqcap_j D, C \sqcup_j D, \exists_j R.C, \forall_j R.C, \tag{1}$$

where $(j,i) \in E$, $C, D \in \widehat{\mathcal{C}}_i \cap \widehat{\mathcal{C}}_j$ and $R \in \widehat{\mathcal{R}}_i \cap \widehat{\mathcal{R}}_j$.

Using the concepts and roles of F-$\mathcal{ALCI}$, we define its formulas, as follows:

Definition 2 (Formulas) *The i-**formulas** are expressions of the form $C \sqsubseteq D$, with $C, D \in \widehat{\mathcal{C}}_i$, for all $i \in V$.*

An F-$\mathcal{ALCI}$-**TBox** or **TBox** is a collection $T = \{T_i\}_{i \in V}$, where T_i is a finite set of i-formulas, for all $i \in V$, called the i-**TBox**. Since, in this paper, we do not consider RBoxes or ABoxes, the terms *TBox*, *ontology* and *knowledge base* will be used interchangeably.

We use, for every $i \in V$, the notation $\overline{\mathcal{R}}_i$ and $\overline{\mathcal{C}}_i$ to denote the set of i-roles and of i-concepts, respectively, that occur in T_i. $\overline{\mathcal{C}}_i$ is a finite subset of $\widehat{\mathcal{C}}_i$, for every $i \in V$. A role name in $\mathcal{R}_j \cap \overline{\mathcal{R}}_i$ or a concept name in $\mathcal{C}_j \cap \overline{\mathcal{C}}_i$ is said to be **imported from** module j **to** module i. Furthermore, since $\overline{\mathcal{C}}_i \subseteq \widehat{\mathcal{C}}_i$, it is obvious that a module i is allowed to use logical connectives subscripted by the index of a module j, whenever $(j,i) \in E$.

2.2 The Semantics

In this subsection, we present the semantics for the language F-$\mathcal{ALCI}$.

Definition 3 *An **interpretation** $\mathcal{I} = \langle \{\mathcal{I}_i\}_{i \in V}, \{r_{ij}\}_{(i,j) \in E} \rangle$ consists of a family $\mathcal{I}_i = \langle \Delta^i, \cdot^i \rangle, i \in V$, of **local interpretations**, together with a family of **image domain relations** $r_{ij} \subseteq \Delta^i \times \Delta^j, (i,j) \in E$, such that $r_{ii} = \mathrm{id}_{\Delta^i}$, for all $i \in V$.*

Notation: For a binary relation $r \subseteq \Delta^i \times \Delta^j$, $X \subseteq \Delta^i$ and $S \subseteq \Delta^i \times \Delta^i$, we set

$$r(X) := \{y \in \Delta^j : (\exists x \in X)((x, y) \in r)\},$$

$$r(S) := \{(z, w) \in \Delta^j \times \Delta^j : (\exists (x, y) \in S)((x, z), (y, w) \in r)\}.$$

A local interpretation function $\cdot^i$ interprets i-role names and i-concept names, as well as $\bot_i$ and $\top_i$, as follows:

- $C^i \subseteq \Delta^i$, for all $C \in \mathcal{C}_i$,
- $R^i \subseteq \Delta^i \times \Delta^i$, for all $R \in \mathcal{R}_i$,
- $\top_i^i = \Delta^i$, $\bot_i^i = \emptyset$.

The interpretations of imported role names and imported concept names are computed by the following rules:

- $C^i = r_{ji}(C^j)$, for all $C \in \mathcal{C}_j \cap \widehat{\mathcal{C}}_i$,
- $R^i = r_{ji}(R^j)$, for all $R \in \mathcal{R}_j \cap \widehat{\mathcal{R}}_i$,
- $\top_j^i = r_{ji}(\Delta^j)$, $\bot_j^i = \emptyset$.

The recursive features of the local interpretation function $\cdot^i$ are as follows:

- $R^{-i} = R^{i^-}$, for all $R \in \mathcal{R}_i$,
- $(\neg_j C)^i = r_{ji}(\Delta^j - C^j)$
- $(C \sqcap_j D)^i = r_{ji}(C^j \cap D^j)$
- $(C \sqcup_j D)^i = r_{ji}(C^j \cup D^j)$
- $(\exists_j R.C)^i = r_{ji}(\{x \in \Delta^j : (\exists y)((x, y) \in R^j \text{ and } y \in C^j)\})$
- $(\forall_j R.C)^i = r_{ji}(\{x \in \Delta^j : (\forall y)((x, y) \in R^j \text{ implies } y \in C^j)\})$

For all $i \in V$, i-**satisfiability**, denoted by $\models^i$, is defined by $\mathcal{I} \models^i C \sqsubseteq D$ iff $C^i \subseteq D^i$. Given a TBox $T = \{T_i\}_{i \in V}$, the interpretation $\mathcal{I}$ is a **model** of T_i, written $\mathcal{I} \models^i T_i$, iff $\mathcal{I} \models^i \tau$, for every $\tau \in T_i$. Moreover, $\mathcal{I}$ is a **model** of T, written $\mathcal{I} \models T$, iff $\mathcal{I} \models^i T_i$, for every $i \in V$.

Let $w \in V$. Define $G_w = \langle V_w, E_w \rangle$ to be the subgraph of G induced by those vertices in G from which w is reachable and $T_w^* := \{T_i\}_{i \in V_w}$. We say that an F-$\mathcal{ALCI}$-ontology $T = \{T_i\}_{i \in V}$ is **consistent as witnessed by a module** T_w if T_w^* has a model $\mathcal{I} = \langle \{\mathcal{I}_i\}_{i \in V_w}, \{r_{ij}\}_{(i,j) \in E_w} \rangle$, such that $\Delta^w \neq \emptyset$. A concept C is **satisfiable as witnessed by** T_w if there is a model $\mathcal{I}$ of T_w^*, such that $C^w \neq \emptyset$. A concept subsumption $C \sqsubseteq D$ **is valid as witnessed by** T_w, denoted by $C \sqsubseteq_w D$, if, for every model $\mathcal{I}$ of T_w^*, $C^w \subseteq D^w$. An alternative notation for $C \sqsubseteq_w D$ is $T_w^* \models_w C \sqsubseteq D$.

Examples: 1. This example illustrates a feature of the syntax and semantics of contextualized intersection in F-$\mathcal{ALCI}$. (See Figure 1.) Suppose that in Module i, there are two concepts, named A and B, corresponding, respectively, to employees of a company with salaries less than \$100,000 and greater than or equal to \$100,000. Module j, on the other hand, has one native concept C corresponding to categories of employees in the company, e.g., administrators, managers, directors, clerks, etc., and imports concepts A and B from module i.

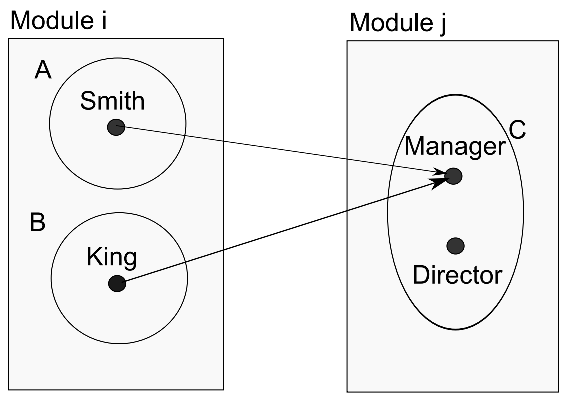

Fig. 1. Interpretation of Contextualized Intersection.

The image domain relation r_{ij} maps both manager `Smith`, who earns a salary of less than \$100,000 and senior manager `King`, who earns a salary of more than \$100,000 to `Manager`, that belongs to concept C in j. One may verify that, whereas $(A \sqcap_i B)^j = \emptyset$, we have that $(A \sqcap_j B)^j \neq \emptyset$. As an explanation, note that the interpretation $(A \sqcap_i B)^j$ is asking about employees that are earning at the same time less than \$100,000 and at least \$100,000. Clearly, no such employees exist. On the other hand, the interpretation of $(A \sqcap_j B)^j$ tries to classify those categories of employees that contain both individuals of high and individuals of lower salaries. `Manager` is obviously such a category.
2. This second example illustrates a feature of the syntax and semantics of contextualized negation in F-$\mathcal{ALCI}$. (See Figure 2.) We deal again with two modules i and j and with the same concepts as before. In this example we assume that all individuals in the universe are shown in the picture. Concept A contains two employees `Smith` and `Jones`, whereas Concept B contains two employees `King` and `Prince`. Both `Smith` and `King` are managers and both `Jones` and `Prince` are directors. This is reflected in the image domain relation r_{ij}, as illustrated in Figure 2. One may verify that, in this case, $(\neg_i A)^j = C^j$, whereas $(\neg_j A)^j = \emptyset$. This formal semantical interpretation may be explained by pointing out that from the point of view of Module j, the interpretation $(\neg_i A)^j$ contains those categories of employees that contain individuals earning at least \$100,000. On the other hand, again from the point of view of Module j, the interpretation of $(\neg_j A)^j$ refers to those categories of employees that do not contain any individual with lower salary. □

3 The Property of Exactness and a Characterization for F-$\mathcal{ALCI}$

Exactness is a property of *some* interpretations of federated description logics, which ensures seamless propagation of knowledge across importing chains. More

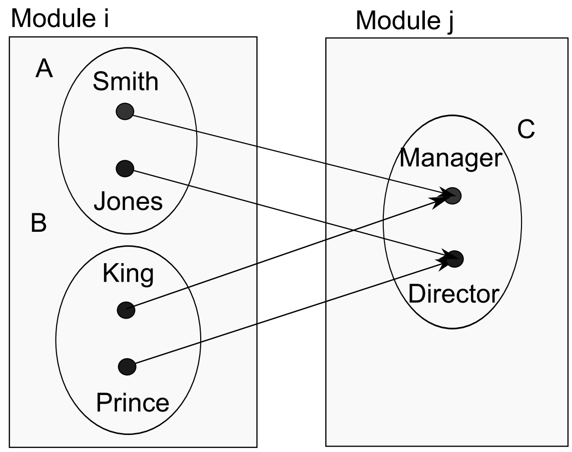

Fig. 2. Interpretation of Contextualized Negation.

precisely, if a concept C in module k is imported by both module i and module j, and module j imports module i, then exactness is equivalent to $r_{kj}(C^k) = r_{ij}(r_{ki}(C^k))$. This has the consequence that, if $\mathcal{I} \models^i C \sqsubseteq D$, then $\mathcal{I} \models^j C \sqsubseteq D$. This is a property that may be very desirable in some contexts but not absolutely necessary in others. Since it imposes rather strong restrictions on the models, we impose it on our interpretations selectively rather than require that it holds universally, as is done in [5].

Definition 4 (Exactness) *Given $(i, j) \in E$, an F-$\mathcal{ALCI}$-interpretation $\mathcal{I} = \langle \{\mathcal{I}_i\}_{i \in V}, \{r_{ij}\}_{(i,j) \in E} \rangle$ is said to be (i, j)-***exact*** *if, for every $C \in \widehat{\mathcal{C}}_i \cap \widehat{\mathcal{C}}_j$, $r_{ij}(C^i) = C^j$. $\mathcal{I}$ is* **exact** *if it is (i, j)-exact, for all $(i, j) \in E$.*

Example: Figure 3 depicts an F-$\mathcal{ALCI}$-interpretation that is not exact. The graph G has three vertices i, j, k and three edges $(k, i), (k, j), (i, j)$. There is one k-concept name A that is imported by both modules i and j and there are no role names. Note that $A^i = r_{ki}(A^k)$ and $A^j = r_{kj}(A^k)$, as required by the definition of interpretation. However, for the concept $\neg_k A \in \widehat{\mathcal{C}}_i \cap \widehat{\mathcal{C}}_j$, we obtain $r_{ij}((\neg_k A)^i) = r_{ij}(r_{ki}(\Delta^k \backslash A^k)) = r_{ij}(r_{ki}(\{y\})) = \emptyset$, whereas $(\neg_k A)^j = r_{kj}(\Delta^k \backslash A^k) = r_{kj}(\{y\}) = \{y''\}$. Thus, the indicated interpretation is not an exact interpretation. $\square$

Note that, in general, the notion of exactness in Definition 4 requires that the condition $r_{ij}(C^i) = C^j$ holds for an infinite collection of concept expressions. For our applications the following weaker concept of exactness, that depends on the contents of a specific knowledge base under consideration, suffices. First let us call a set $\mathcal{E}_i \subseteq \widehat{\mathcal{C}}_i$ of i-concept expressions **closed** if it is closed under concept sub-expressions, i.e., for every $C \in \mathcal{E}_i$, all sub-concepts of C are also in $\mathcal{E}_i$.

Definition 5 (Exactness for T**)** *Let $\mathcal{E} = \{\mathcal{E}_i\}_{i \in V}$, with $\mathcal{E}_i \subseteq \widehat{\mathcal{C}}_i$, $i \in V$, be a V-indexed collection of closed sets of concept expressions and $\mathcal{I} = \langle \{\mathcal{I}_i\}_{i \in V}$,*

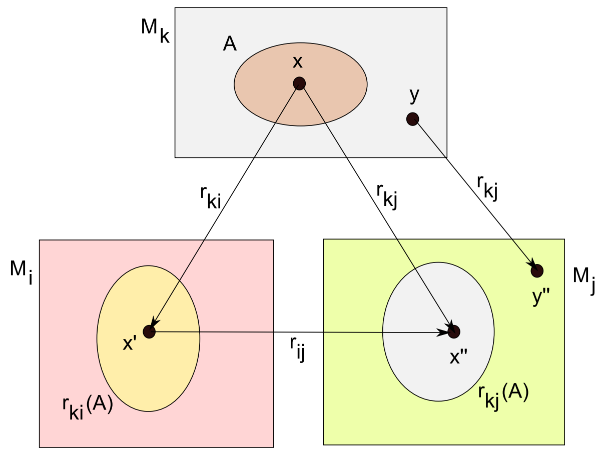

Fig. 3. A Non-exact Interpretation.

*$\{r_{ij}\}_{(i,j)\in E}\rangle$ be an F-$\mathcal{ALCI}$-interpretation. Given $(i,j) \in E$, $\mathcal{I}$ is said to be (i,j)-***exact for** *$\mathcal{E}$ if, for every $C \in \mathcal{E}_i \cap \mathcal{E}_j$, $r_{ij}(C^i) = C^j$. $\mathcal{I}$ is* **exact for** *$\mathcal{E}$ if it is (i,j)-exact for $\mathcal{E}$, for all $(i,j) \in E$.*

*Let $T = \{T_i\}_{i\in V}$ be an F-$\mathcal{ALCI}$-ontology and $\mathcal{I} = \langle\{\mathcal{I}_i\}_{i\in V}, \{r_{ij}\}_{(i,j)\in E}\rangle$ an F-$\mathcal{ALCI}$-interpretation. $\mathcal{I}$ is said to be (i,j)-***exact for** *T if it is (i,j)-exact for $\overline{\mathcal{C}} := \{\overline{\mathcal{C}}_i\}_{i\in V}$ and it is said to be* **exact for** *T if it is exact for $\overline{\mathcal{C}} := \{\overline{\mathcal{C}}_i\}_{i\in V}$.*

An alternative condition characterizing the exactness of an F-$\mathcal{ALCI}$-interpretation is provided in the following lemma.

Lemma 6 *An F-$\mathcal{ALCI}$-interpretation $\mathcal{I} = \langle\{\mathcal{I}_i\}_{i\in V}, \{r_{ij}\}_{(i,j)\in E}\rangle$ is exact if and only if, for all $k, i, j \in V$, such that $(k,i), (k,j)$, $(i,j) \in E$, $r_{ij}(r_{ki}(C^k)) = r_{kj}(C^k)$, for every $C \in \widehat{\mathcal{C}}_i \cap \widehat{\mathcal{C}}_j \cap \widehat{\mathcal{C}}_k$. The importing relations are depicted in the following importing diagram.*

$$\begin{tikzcd} & k \arrow[dl, "r_{ki}"'] \arrow[dr, "r_{kj}"] & \\ i \arrow[rr, "r_{ij}"'] & & j \end{tikzcd}$$

Proof:

- $\Rightarrow$: If $\mathcal{I}$ is exact, then, for every $C \in \widehat{\mathcal{C}}_i \cap \widehat{\mathcal{C}}_j \cap \widehat{\mathcal{C}}_k$, we have that $r_{ij}(r_{ki}(C^k)) = r_{ij}(C^i) = C^j = r_{kj}(C^k)$.
- $\Leftarrow$: For this direction, suppose that, for all $k, i, j \in V$, such that $(k,i), (k,j)$, $(i,j) \in E$, $r_{ij}(r_{ki}(B^k)) = r_{kj}(B^k)$, for every i-,j- and k-concept B. Consider $C \in \widehat{\mathcal{C}}_i \cap \widehat{\mathcal{C}}_j$. To show that $r_{ij}(C^i) = C^j$, we apply structural induction on C. We have:
  - $r_{ij}(\top^i_k) = r_{ij}(r_{ki}(\Delta^k)) = r_{kj}(\Delta^k) = \top^j_k$.

- $r_{ij}(C^i) = r_{ij}(r_{ki}(C^k)) = r_{kj}(C^k) = C^j$, for every $C \in \mathcal{C}_k \cap \widehat{\mathcal{C}}_i \cap \widehat{\mathcal{C}}_j$.
- $r_{ij}((\neg_k C)^i) = r_{ij}(r_{ki}(\Delta^k - C^k)) = r_{ij}(r_{ki}((\neg_k C)^k)) = r_{kj}((\neg_k C)^k) = r_{kj}(\Delta^k - C^k) = (\neg_k C)^j$.
- $r_{ij}((C \sqcap_k D)^i) = r_{ij}(r_{ki}(C^k \cap D^k)) = r_{ij}(r_{ki}((C \sqcap_k D)^k)) = r_{kj}((C \sqcap_k D)^k) = r_{kj}(C^k \cap D^k) = (C \sqcap_k D)^j$.
- $r_{ij}((\exists_k R.C)^i) = r_{ij}(r_{ki}((\exists_k R.C)^k)) = r_{kj}((\exists_k R.C)^k) = (\exists_k R.C)^j$.

The cases of $C \sqcup_k D$ and of $\forall_k R.C$ are handled similarly. ■

Employing the same proof, but with "exact for $\mathcal{E}$" in place of "exact", we obtain the following lemma providing a necessary and sufficient condition for the exactness of an F-$\mathcal{ALCI}$-interpretation for a given V-indexed collection $\mathcal{E}$ of closed sets of concept expressions.

Lemma 7 *Let $\mathcal{E} = \{\mathcal{E}_i\}_{i \in V}$, with $\mathcal{E}_i \subseteq \widehat{\mathcal{C}}_i$, $i \in V$, be a V-indexed collection of closed sets of concept expressions and $\mathcal{I} = \langle \{\mathcal{I}_i\}_{i \in V}, \{r_{ij}\}_{(i,j) \in E} \rangle$ an F-$\mathcal{ALCI}$-interpretation. $\mathcal{I}$ is exact for $\mathcal{E}$ if and only if, for all $k, i, j \in V$, such that $(k,i), (k,j), (i,j) \in E$, $r_{ij}(r_{ki}(C^k)) = r_{kj}(C^k)$, for every $C \in \mathcal{E}_i \cap \mathcal{E}_j \cap \mathcal{E}_k$. The importing relations are depicted in the following importing diagram.*

$$\begin{tikzcd} & k \arrow[dl, "r_{ki}"'] \arrow[dr, "r_{kj}"] & \\ i \arrow[rr, "r_{ij}"'] & & j \end{tikzcd}$$

Based on the definition of an exact interpretation, we define exact models of an F-$\mathcal{ALCI}$-ontology.

Definition 8 (Exact Model) *Let $T = \{T_i\}_{i \in V}$ be an F-$\mathcal{ALCI}$-ontology. An interpretation $\mathcal{I} = \langle \{\mathcal{I}_i\}_{(i,w) \in E}, \{r_{ij}\}_{(i,j),(j,w) \in E} \rangle$ is an* **exact model** *of T if it is exact for T and $\mathcal{I} \models T$. T is said to be* **exactly consistent as witnessed by a module** *T_w if there exists an exact model $\mathcal{I}$ of T_w^*, such that $\Delta^w \neq \emptyset$. A concept C is* **exactly satisfiable as witnessed by** *T_w if there exists an exact model $\mathcal{I}$ of T_w^*, such that $C^w \neq \emptyset$. Finally, a concept subsumption $C \sqsubseteq D$ is* **exactly valid as witnessed by** *T_w, denoted $C \sqsubseteq_w^e D$ if, for every exact model $\mathcal{I}$ of T_w^*, $C^w \subseteq D^w$. In this case we also write $T_w^* \models_w^e C \sqsubseteq D$.*

4 A Reduction from F-$\mathcal{ALCI}$ to $\mathcal{ALCI}$

A **reduction** $\mathfrak{R}$ from an F-$\mathcal{ALCI}$ KB $\Sigma_d = \{T_i\}$ to an $\mathcal{ALCI}$ KB $\Sigma := \mathfrak{R}(\Sigma_d)$ is obtained as follows:

The signature of Σ is the union of the local signatures of the modules together with a global top $\top$, a global bottom $\bot$, local top concepts $\top_i$, for all $i \in V$, and, finally, a collection of new role names $\{R_{ij}\}_{(i,j) \in E}$, i.e.,

$$\mathsf{Sig}(\Sigma) = \bigcup_i (\mathcal{C}_i \cup \mathcal{R}_i) \cup \{\top, \bot\} \cup \{\top_i : 1 \leq i \leq n\} \cup \{R_{ij} : (i,j) \in E\}.$$

Moreover, various axioms derived from the structure of Σ_d are added to Σ.

– For each $C \in \mathcal{C}_i$, $C \sqsubseteq \top_i$ is added to Σ.
– For each $R \in \mathcal{R}_i$, $\top_i$ is stipulated to be the domain and range of R, i.e., $\top \sqsubseteq \forall R^-.\top_i$ and $\top \sqsubseteq \forall R.\top_i$ are added to Σ.
– For each new role name R_{ij}, $\top_i$ is stipulated to be its domain and $\top_j$ to be its range, i.e., $\top \sqsubseteq \forall R_{ij}^-.\top_i$ and $\top \sqsubseteq \forall R_{ij}.\top_j$ are added to Σ.
– For each $C \sqsubseteq D \in T_i$, $\#_i(C) \sqsubseteq \#_i(D)$ is added to Σ, where $\#_i$ is a function from $\widehat{\mathcal{C}}_i$ to the set of $\mathcal{ALCI}$-concepts. The precise definition of $\#_i$ is given below.

If, in addition to the previous conditions, for every importing diagram of the form

$$\begin{tikzcd} & k \arrow[dl] \arrow[dr] & \\ i \arrow[rr] & & j \end{tikzcd} \tag{2}$$

and all $C \in \overline{\mathcal{C}}_i \cap \overline{\mathcal{C}}_j \cap \overline{\mathcal{C}}_k$, $\exists R_{ij}^-.(\exists R_{ki}^-.\#_k(C)) = \exists R_{kj}^-.\#_k(C)$ is added to Σ, then the reduction is said to be an **exact reduction** and is denoted by $\mathfrak{R}_e(\Sigma_d)$.

The mapping $\#_i(C)$ serves to maintain the compatibility of the concept domains. It is defined by induction on the structure of $C \in \widehat{\mathcal{C}}_i$:

– $\#_i(C) = C$, if $C \in \mathcal{C}_i$;
– $\#_i(C) = \exists R_{ji}^-.\#_j(C)$, if $C \in \mathcal{C}_j \cap \widehat{\mathcal{C}}_i$;
– $\#_i(\neg_j D) = \exists R_{ji}^-.(\neg\, \#_j(D) \sqcap \top_j)$, if $C = \neg_j D$;
– $\#_i(D \boxplus_j E) = \exists R_{ji}^-.(\#_j(D) \boxplus \#_j(E))$, if $C = D \boxplus_j E$, where $\boxplus = \sqcap$ or $\boxplus = \sqcup$;
– $\#_i(\exists_j R.D) = \exists R_{ji}^-.(\exists R_{kj}^-.(\exists R.(\exists R_{kj}.\#_j(D))))$, if $C = \exists_j R.D$, with $R \in \mathcal{R}_k$ or $R \in \mathcal{R}_k^- := \{R^- : R \in \mathcal{R}_k\}$;
– $\#_i(\forall_j R.D) = \exists R_{ji}^-.(\forall R_{kj}^-.(\forall R.(\forall R_{kj}.\#_j(D))))$, if $C = \forall_j R.D$, with $R \in \mathcal{R}_k \cup \mathcal{R}_k^-$.

It will be shown that the reduction $\mathfrak{R}$ is sound and complete in the sense that, if the local top concept $\top_w$ in $\mathfrak{R}(\Sigma_d)$, that corresponds to a module T_w in Σ_d, is satisfiable in an $\mathcal{ALCI}$-model of $\mathfrak{R}(\Sigma_d)$, then Σ_d itself is consistent as witnessed by T_w and vice-versa. Soundness will be taken up in the next section and completeness in Section 6.

5 Soundness of the Reduction

Definition 9 *Let $\Sigma_d = \{T_i\}_{i \in V}$ be an F-$\mathcal{ALCI}$ KB and $\mathcal{I} = \langle \Delta^{\mathcal{I}}, \cdot^{\mathcal{I}} \rangle$ an interpretation of the $\mathcal{ALCI}$ KB $\mathfrak{R}(\Sigma_d)$. Construct an interpretation $\mathtt{F}(\mathcal{I}) = \langle \{\mathcal{I}_i\}_{i \in V}, \{r_{ij}\}_{(i,j) \in E} \rangle$ for Σ_d as follows:*

– *$\Delta^i = \top_i^{\mathcal{I}}$, for all $i \in V$;*
– *$C^i = C^{\mathcal{I}}$, for every $C \in \mathcal{C}_i$;*
– *$R^i = R^{\mathcal{I}}$, for every $R \in \mathcal{R}_i$;*
– *$r_{ij} = R_{ij}^{\mathcal{I}}$, for every $(i,j) \in E$.*

We start with a technical lemma that shows, roughly speaking, that the image of the interpretation of a concept C under the interpretation of one of the new role names R_{ij} is equal to the interpretation in the same model of the concept $\exists R^{-}_{ij}.C$. This lemma is preparatory in dealing with the various cases involved in the definition of the translation function $\#_i$.

Lemma 10 *Let Σ_d be an F-$\mathcal{ALCI}$* KB *and $\mathcal{I} = \langle \Delta^{\mathcal{I}}, \cdot^{\mathcal{I}} \rangle$ an interpretation for $\mathfrak{R}(\Sigma_d)$. Then, for every concept $C \in \widehat{\mathcal{C}}_i$, such that $\exists R^{-}_{ij}.C$ occurs in $\mathfrak{R}(\Sigma_d)$,*

$$R^{\mathcal{I}}_{ij}(C^{\mathcal{I}}) = (\exists R^{-}_{ij}.C)^{\mathcal{I}}.$$

Proof:
We do indeed have

$$\begin{aligned}(\exists R^{-}_{ij}.C)^{\mathcal{I}} &= \{x \in \Delta^{\mathcal{I}} : (\exists y \in C^{\mathcal{I}})((x,y) \in R^{\mathcal{I}\,-}_{ij})\} \quad \text{(by the definition of } \cdot^{\mathcal{I}}) \\ &= \{x \in \Delta^{\mathcal{I}} : (\exists y \in C^{\mathcal{I}})((y,x) \in R^{\mathcal{I}}_{ij})\} \quad \text{(by the definition of } R^{\mathcal{I}\,-}_{ij}) \\ &= R^{\mathcal{I}}_{ij}(C^{\mathcal{I}}). \quad \text{(by the definition of } R^{\mathcal{I}}_{ij}(C^{\mathcal{I}}))\end{aligned}$$

■

Next, we present another technical lemma to the effect that the interpretation of the concept $\forall R^{-}_{kj}.(\forall R.(\forall R_{kj}.\#_j(C)))$ formed using the translation $\#_j(C)$ of a concept $C \in \widehat{\mathcal{C}}_j$ and the role name $R \in \mathcal{R}_k$, equals to

$$\forall R^{\mathcal{I}}_{kj}(R^{\mathcal{I}}).\#_j(C)^{\mathcal{I}} := \{x \in \top^{\mathcal{I}}_j : (\forall y \in \top^{\mathcal{I}}_j)((x,y) \in R^{\mathcal{I}}_{kj}(R^{\mathcal{I}}) \to y \in \#_j(C)^{\mathcal{I}})\}.$$

This lemma will help us deal with the universal quantification case involved in the recursive definition of the translation function $\#_i$.

Lemma 11 *Let Σ_d be an F-$\mathcal{ALCI}$* KB *and $\mathcal{I} = \langle \Delta^{\mathcal{I}}, \cdot^{\mathcal{I}} \rangle$ an interpretation for $\mathfrak{R}(\Sigma_d)$. Then, for all $C \in \widehat{\mathcal{C}}_j$, $R \in \mathcal{R}_k$, such that $\forall R^{-}_{kj}.(\forall R.(\forall R_{kj}.\#_j(C)))$ occurs in $\mathfrak{R}(\Sigma_d)$*

$$\forall R^{-}_{kj}.(\forall R.(\forall R_{kj}.\#_j(C)))^{\mathcal{I}} = \forall R^{\mathcal{I}}_{kj}(R^{\mathcal{I}}).\#_j(C)^{\mathcal{I}}.$$

Proof:
For the left-to-right inclusion, suppose that $x \in \forall R^{-}_{kj}.(\forall R.(\forall R_{kj}.\#_j(C)))^{\mathcal{I}}$. The following diagrams help illustrate the argument.

$$\begin{tikzcd} x & y \arrow[l, "R^{\mathcal{I}}_{kj}"'] \arrow[d, "R^{\mathcal{I}}"] \\ w & z \arrow[l, "R^{\mathcal{I}}_{kj}"] \end{tikzcd} \qquad \begin{tikzcd} x & u \arrow[l, "R^{\mathcal{I}}_{kj}"'] \arrow[d, "R^{\mathcal{I}}"] \\ v & t \arrow[l, "R^{\mathcal{I}}_{kj}"] \end{tikzcd}$$

Then, for all $y \in \Delta^{\mathcal{I}}$, with $(y,x) \in R^{\mathcal{I}}_{kj}$, we must have $y \in \forall R.(\forall R_{kj}.\#_j(C))^{\mathcal{I}}$. Thus, for all $z \in \Delta^{\mathcal{I}}$, such that $(y,z) \in R^{\mathcal{I}}$, we have that $z \in \forall R_{kj}.\#_j(C)^{\mathcal{I}}$, i.e., for all $w \in \Delta^{\mathcal{I}}$, such that $(z,w) \in R^{\mathcal{I}}_{kj}$, $w \in \#_j(C)^{\mathcal{I}}$.

Now assume that $(x,v) \in R^{\mathcal{I}}_{kj}(R^{\mathcal{I}})$, for some $v \in \Delta^{\mathcal{I}}$. Then, there exist $u,t \in \Delta^{\mathcal{I}}$, such that $(u,t) \in R^{\mathcal{I}}$, $(u,x),(t,v) \in R^{\mathcal{I}}_{kj}$. Then, by what was shown in the previous paragraph, $v \in \#_j(C)^{\mathcal{I}}$, whence we conclude that $x \in \forall R_{kj}^{\mathcal{I}}(R^{\mathcal{I}}).\#_j(C)^{\mathcal{I}}$.

For the right-to-left inclusion, assume that $x \in \forall R_{kj}^{\mathcal{I}}(R^{\mathcal{I}}).\#_j(C)^{\mathcal{I}}$. Thus, for all $v \in \Delta^{\mathcal{I}}$, such that $(x, v) \in R_{kj}^{\mathcal{I}}(R^{\mathcal{I}})$, we must have $v \in \#_j(C)^{\mathcal{I}}$. Now assume that $(y, x) \in R_{kj}^{\mathcal{I}}$, $(y, z) \in R^{\mathcal{I}}$ and $(z, w) \in R_{kj}^{\mathcal{I}}$. This implies that $(x, w) \in R_{kj}^{\mathcal{I}}(R^{\mathcal{I}})$. Thus, $w \in \#_j(C)^{\mathcal{I}}$. This proves that $x \in \forall R_{kj}^{-}.(\forall R.(\forall R_{kj}.\#_j(C)))^{\mathcal{I}}$.

■

To connect the interpretation $\mathcal{I}$ with its federated counterpart $\mathtt{F}(\mathcal{I})$, we need to establish a correspondence between the interpretation of the translation $\#_i(C)$ of a concept $C \in \widehat{\mathcal{C}}_i$ under $\mathcal{I}$ and that of the concept C under $\mathtt{F}(\mathcal{I})$. This relationship is explored in the following lemma.

Lemma 12 *Let Σ_d be an F-$\mathcal{ALCI}$* KB*, $\mathcal{I} = \langle \Delta^{\mathcal{I}}, \cdot^{\mathcal{I}} \rangle$ an interpretation for $\mathfrak{R}(\Sigma_d)$ and $\mathtt{F}(\mathcal{I}) = \langle \{\mathcal{I}_i\}_{i \in V}, \{r_{ij}\}_{(i,j) \in E} \rangle$, with $\mathcal{I}_i = \langle \Delta^i, \cdot^i \rangle, i \in V$. Then*

$$\#_i(C)^{\mathcal{I}} = C^i, \quad \textit{for every } C \in \widehat{\mathcal{C}}_i,\ i \in V.$$

Proof:

We do this by structural induction on C.

For the basis of the induction, if $C \in \mathcal{C}_i$,

$$\begin{aligned} \#_i(C)^{\mathcal{I}} &= C^{\mathcal{I}} \quad \text{(by the definition of } \#_i(C)) \\ &= C^i, \quad \text{(by the definition of } C^i) \end{aligned}$$

whereas, if $C \in \mathcal{C}_j \cap \widehat{\mathcal{C}}_i$,

$$\begin{aligned} \#_i(C)^{\mathcal{I}} &= (\exists R_{ji}^{-}.\#_j(C))^{\mathcal{I}} \quad \text{(by the definition of } \#_i(C)) \\ &= R_{ji}^{\mathcal{I}}(\#_j(C)^{\mathcal{I}}) \quad \text{(by Lemma 10)} \\ &= r_{ji}(C^j) \quad \text{(by the definition of } r_{ji} \text{ and the previous case)} \\ &= C^i. \quad \text{(by the definition of } C^i) \end{aligned}$$

For $C = \neg_j D$, we have

$$\begin{aligned} \#_i(\neg_j D)^{\mathcal{I}} &= (\exists R_{ji}^{-}.(\neg \#_j(D) \sqcap \top_j))^{\mathcal{I}} \quad \text{(by the definition of } \#_i(\neg_j D)) \\ &= R_{ji}^{\mathcal{I}}((\neg \#_j(D) \sqcap \top_j)^{\mathcal{I}}) \quad \text{(by Lemma 10)} \\ &= R_{ji}^{\mathcal{I}}((\neg \#_j(D))^{\mathcal{I}} \cap \top_j^{\mathcal{I}}) \quad \text{(by the definition of } \cdot^{\mathcal{I}}) \\ &= R_{ji}^{\mathcal{I}}((\Delta^{\mathcal{I}} \backslash \#_j(D)^{\mathcal{I}}) \cap \top_j^{\mathcal{I}}) \quad \text{(by the definition of } \cdot^{\mathcal{I}}) \\ &= r_{ji}((\Delta^{\mathcal{I}} \backslash D^j) \cap \Delta^j) \quad \text{(by the definition of } \mathtt{F}(\mathcal{I}) \\ &\qquad\qquad \text{and the induction hypothesis)} \\ &= r_{ji}(\Delta^j \backslash D^j) \quad \text{(set-theoretically)} \\ &= (\neg_j D)^i. \quad \text{(by the defintiion of } (\neg_j D)^i) \end{aligned}$$

For $\boxplus = \sqcap$ or $\boxplus = \sqcup$, and denoting by $\oplus = \cap$ or $\oplus = \cup$, respectively, the corresponding set-theoretic operation,

$$\begin{aligned}
\#_i(D \boxplus_j E)^{\mathcal{I}} &= (\exists R_{ji}^{-}.(\#_j(D) \boxplus \#_j(E)))^{\mathcal{I}} \quad \text{(by the definition of } \#_i(D \boxplus_j E)^{\mathcal{I}})\\
&= R_{ji}^{\mathcal{I}}((\#_j(D) \boxplus \#_j(E))^{\mathcal{I}}) \quad \text{(by Lemma 10)}\\
&= R_{ji}^{\mathcal{I}}(\#_j(D)^{\mathcal{I}} \oplus \#_j(E)^{\mathcal{I}}) \quad \text{(by the definition of } \cdot^{\mathcal{I}})\\
&= r_{ji}(D^j \oplus E^j) \quad \text{(by the definition of } \mathtt{F}(\mathcal{I}) \text{ and the induction hypothesis)}\\
&= (D \boxplus_j E)^i. \quad \text{(by the definition of } (D \boxplus_j E)^i)
\end{aligned}$$

For $C = \#_i(\exists_j R.D)$, with $R \in \mathcal{R}_k$, we first show that

$$r_{kj}(R^{\mathcal{I}-}(r_{kj}^{-}(D^j))) = \{x \in \Delta^j : (\exists w \in D^j)((x,w) \in R^j)\}. \tag{3}$$

For the left-to-right inclusion, assume that $x \in r_{kj}(R^{\mathcal{I}-}(r_{kj}^{-}(D^j)))$. Then, there exists $y \in R^{\mathcal{I}-}(r_{kj}^{-}(D^j))$, such that $(y,x) \in r_{kj}$. Thus, there exists $z \in r_{kj}^{-}(D^j)$, such that $(y,z) \in R^{\mathcal{I}}$. Hence, there exists $w \in D^j$, such that $(z,w) \in r_{kj}$. These relations are depicted in the following diagram.

$$\begin{tikzcd}
y \arrow[r, "r_{kj}"] \arrow[d, "R^{\mathcal{I}}"'] & x \\
z \arrow[r, "r_{kj}"'] & w
\end{tikzcd}$$

This shows that $(x,w) \in r_{kj}(R^{\mathcal{I}}) = r_{kj}(R^k) = R^j$ and, as a result, that $x \in \{t \in \Delta^j : (\exists w \in D^j)((t,w) \in R^j)\}$.

Suppose, for the reverse inclusion, that $x \in \{t \in \Delta^j : (\exists w \in D^j)((t,w) \in R^j)\}$. Thus, there exists $w \in D^j$, such that $(x,w) \in R^j = r_{kj}(R^k) = r_{kj}(R^{\mathcal{I}})$. Hence, there exists $(y,z) \in R^{\mathcal{I}}$, such that $(y,x) \in r_{kj}$ and $(w,z) \in r_{kj}^{-}$. This shows that $x \in r_{kj}(y) \subseteq r_{kj}(R^{\mathcal{I}^-}(z)) \subseteq r_{kj}(R^{\mathcal{I}^-}(r_{kj}^{-}(w))) \subseteq r_{kj}(R^{\mathcal{I}^-}(r_{kj}^{-}(D^j)))$ and concludes the proof of Equation (3).

Now we get that

$$\begin{aligned}
\#_i(\exists_j R.D)^{\mathcal{I}} &= (\exists R_{ji}^{-}.(\exists R_{kj}^{-}.(\exists R.(\exists R_{kj}.(\#_j(D))))))^{\mathcal{I}}\\
&\quad \text{(by the definition of } \#_i(\exists_j R.D))\\
&= R_{ji}^{\mathcal{I}}(R_{kj}^{\mathcal{I}}(R^{\mathcal{I}-}(R_{kj}^{\mathcal{I}-}(\#_j(D)^{\mathcal{I}})))) \quad \text{(by Lemma 10)}\\
&= r_{ji}(r_{kj}(R^{\mathcal{I}-}(r_{kj}^{-}(D^j)))) \quad \text{(by the definition of } \mathtt{F}(\mathcal{I}) \text{ and the induction hypothesis)}\\
&= r_{ji}(\{x \in \Delta^j : (\exists y \in D^j)((x,y) \in R^j)\}) \quad \text{(by Equation (3))}\\
&= (\exists_j R.D)^i. \quad \text{(by the definition of } (\exists_j R.D)^i)
\end{aligned}$$

For $C = \#_i(\forall_j R.D)$, with $R \in \mathcal{R}_k$, recall that, by Lemma 11,

$$\begin{aligned}
&(\forall {R_{kj}}^{-}.(\forall R.(\forall R_{kj}.(\#_j(D)))))^{\mathcal{I}} =\\
&\qquad \{x \in \top_j^{\mathcal{I}} : (\forall y \in \top_j^{\mathcal{I}})((x,y) \in R_{kj}^{\mathcal{I}}(R^{\mathcal{I}}) \to y \in \#_j(D)^{\mathcal{I}})\}.
\end{aligned} \tag{4}$$

Therefore, we have

$$\begin{aligned}
\#_i(\forall_j R.D)^{\mathcal{I}} &= (\exists R_{ji}^{-}.(\forall R_{kj}^{-}.(\forall R.(\forall R_{kj}.(\#_j(D))))))^{\mathcal{I}} \\
&\quad \text{(by the definition of } \#_i(\forall_j R.D)) \\
&= R_{ji}^{\mathcal{I}}((\forall R_{kj}^{-}(\forall R.(\forall R_{kj}.(\#_j(D)))))^{\mathcal{I}}) \quad \text{(by Lemma 10)} \\
&= R_{ji}^{\mathcal{I}}(\{x \in \top_j^{\mathcal{I}} : (\forall y \in \top_j^{\mathcal{I}})((x,y) \in R_{kj}^{\mathcal{I}}(R^{\mathcal{I}}) \to y \in \#_j(D)^{\mathcal{I}})\}) \\
&\quad \text{(by Equation (4))} \\
&= r_{ji}(\{x \in \Delta^j : (\forall y \in \Delta^j)((x,y) \in R^j \to y \in D^j)\}) \\
&\quad \text{(by the definition of } \mathtt{F}(\mathcal{I}) \text{ and the induction hypothesis)} \\
&= (\forall_j R.D)^i. \quad \text{(by the definition of } (\forall_j R.D)^i)
\end{aligned}$$

■

The following is our main soundness theorem for the reduction $\mathfrak{R}$.

Theorem 13 (Soundness) *Let Σ_d be an F-$\mathcal{ALCI}$ KB, and T_w a module of Σ_d. If $\top_w$ is satisfiable with respect to $\mathfrak{R}(T_w^*)$, then Σ_d is consistent as witnessed by T_w.*

Proof:
Suppose that $\top_w$ is satisfiable with respect to $\mathfrak{R}(T_w^*)$. Then $\mathfrak{R}(T_w^*)$ has a model $\mathcal{I} = \langle \Delta^{\mathcal{I}}, \cdot^{\mathcal{I}} \rangle$, such that $\top_w^{\mathcal{I}} \neq \emptyset$. Our goal is to show that $\mathtt{F}(\mathcal{I}) = \langle \{\mathcal{I}_i\}_{i \in V_w}, \{r_{ij}\}_{(i,j) \in E_w} \rangle$ is a model of T_w^*, such that $\Delta^w \neq \emptyset$.

Clearly, we have $\Delta^w = \top_w^{\mathcal{I}} \neq \emptyset$, by the hypothesis. So it suffices to show that $\mathtt{F}(\mathcal{I})$ is a model of the federated ontology T_w^*, i.e., that it satisfies $\mathcal{I}_i \models T_i$, for every $i \in V_w$. Suppose that $C \sqsubseteq D \in T_i$. By the construction of $\mathfrak{R}(T_w^*)$ and the fact that $\mathcal{I} \models \mathfrak{R}(T_w^*)$, we must have $\#_i(C)^{\mathcal{I}} \subseteq \#_i(D)^{\mathcal{I}}$, whence, by Lemma 12, we obtain that $C^i \subseteq D^i$, showing that $\mathtt{F}(\mathcal{I}) \models T_w^*$. ■

To establish the soundness in the case of an *exact* reduction we need to ensure that the federated model $\mathtt{F}(\mathcal{I})$ obtained by the model $\mathcal{I}$ of $\mathfrak{R}_e(\Sigma_d)$ is an exact model of Σ_d. Preliminary work towards this goal is accomplished in the following lemma.

Lemma 14 *Let Σ_d be an F-$\mathcal{ALCI}$ ontology and $\mathcal{I} = \langle \Delta^{\mathcal{I}}, \cdot^{\mathcal{I}} \rangle$ be a model of $\mathfrak{R}_e(\Sigma_d)$. Then, for every importing diagram*

$$\begin{tikzcd}
 & k \arrow[dl] \arrow[dr] & \\
i \arrow[rr] & & j
\end{tikzcd}$$

and all $C \in \overline{\mathcal{C}}_i \cap \overline{\mathcal{C}}_j \cap \overline{\mathcal{C}}_k$, $r_{ij}(r_{ki}(C^k)) = r_{kj}(C^k)$ holds in $\mathtt{F}(\mathcal{I})$.

Proof:

$$\begin{aligned}
r_{ij}(r_{ki}(C^k)) &= R_{ij}^{\mathcal{I}}(R_{ki}^{\mathcal{I}}(\#_k(C)^{\mathcal{I}})) \quad \text{(by the definition of } \mathtt{F}(\mathcal{I}) \text{ and Lemma 12)} \\
&= (\exists R_{ij}^{-}.(\exists R_{ki}^{-}.(\#_k(C))))^{\mathcal{I}} \quad \text{(by Lemma (10))} \\
&= (\exists R_{kj}^{-}.(\#_k(C)))^{\mathcal{I}} \quad \text{(because } \mathcal{I} \models \mathfrak{R}_e(\Sigma_d)) \\
&= R_{kj}^{\mathcal{I}}(\#_k(C)^{\mathcal{I}}) \quad \text{(by Lemma (10))} \\
&= r_{kj}(C^k). \quad \text{(by the definition of } \mathtt{F}(\mathcal{I}) \text{ and Lemma 12)}
\end{aligned}$$

■

Finally, we formulate and present the main result on the exact soundness of the translation $\mathfrak{R}_e$.

Theorem 15 (Exact Soundness) *Let Σ_d be an F-$\mathcal{ALCI}$* KB*, and T_w a module of Σ_d. If $\top_w$ is satisfiable with respect to $\mathfrak{R}_e(T_w^*)$, then Σ_d is exactly consistent as witnessed by T_w.*

Proof:
Suppose that $\top_w$ is satisfiable with respect to $\mathfrak{R}_e(T_w^*)$. Then $\mathfrak{R}_e(T_w^*)$ has a model $\mathcal{I} = \langle \Delta^{\mathcal{I}}, \cdot^{\mathcal{I}} \rangle$, such that $\top_w^{\mathcal{I}} \neq \emptyset$. Our goal is to show that $\mathtt{F}(\mathcal{I})$ is an exact model of T_w^*, such that $\Delta^w \neq \emptyset$.

As before, we have $\Delta^w = \top_w^{\mathcal{I}} \neq \emptyset$, by the hypothesis. So it suffices to show that $\mathtt{F}(\mathcal{I})$ is an exact model of T_w^*, i.e., that it satisfies the two conditions postulated in Definition 8. This amounts to showing, first, that, for all $k, i, j \in V_w$, such that $(k, i), (k, j), (i, j) \in E_w$, $r_{ij}(r_{ki}(C^k)) = r_{kj}(C^k)$, for every $C \in \overline{\mathcal{C}}_i \cap \overline{\mathcal{C}}_j \cap \overline{\mathcal{C}}_k$, and, second, that $\mathcal{I} \models T_i$, for every $i \in V_w$. The first condition holds by Lemma 14. That $\mathcal{I} \models T_i$, for every $i \in V_w$, may be shown exactly as in the first part of the proof. Thus Σ_d is exactly consistent as witnessed by T_w. ■

6 Completeness of the Reduction

We turn now to the proof of the completeness of the reduction $\mathfrak{R}$. Informally speaking, it will be shown that, if an F-$\mathcal{ALCI}$ KB Σ_d is consistent as witnessed by a module T_w, then the corresponding local top concept $\top_w$ in $\Sigma = \mathfrak{R}(\Sigma_d)$ is satisfiable. Moreover, we will obtain a correspondence between exact consistency of Σ_d as witnessed by T_w and satisfiability of $\top_w$ in $\mathfrak{R}_e(\Sigma_d)$.

Definition 16 *Suppose that Σ_d is an F-$\mathcal{ALCI}$* KB *and that $\mathcal{I}_d = \langle \{\mathcal{I}_i\}_{i \in V}, \{r_{ij}\}_{(i,j) \in E} \rangle$ is a model of Σ_d. Construct an interpretation $\mathcal{I} := \mathtt{G}(\mathcal{I}_d) = \langle \Delta^{\mathcal{I}}, \cdot^{\mathcal{I}} \rangle$ of $\mathfrak{R}(\Sigma_d)$ as follows:*

- $\Delta^{\mathcal{I}} = \bigcup_{i \in V} \Delta^i$;
- $\top_i^{\mathcal{I}} = \Delta^i$, *for every* $i \in V$;
- $C^{\mathcal{I}} = C^i$, *for every* $C \in \mathcal{C}_i$;
- $R^{\mathcal{I}} = R^i$, *for every* $R \in \mathcal{R}_i$;
- $R_{ij}^{\mathcal{I}} = r_{ij}$, *for every* $(i, j) \in E$.

To connect the federated interpretation $\mathcal{I}_d$ with its single module-counterpart $\mathcal{I} := \mathtt{G}(\mathcal{I}_d)$, we need to establish a correspondence between the interpretation of the translation $\#_i(C)$ of a concept $C \in \widehat{\mathcal{C}}_i$ under $\mathtt{F}(\mathcal{I}_d)$ and that of the concept C under $\mathcal{I}$. Such a correspondence is revealed in the following lemma.

Lemma 17 *Let Σ_d be an F-$\mathcal{ALCI}$* KB*, $\mathcal{I}_d = \langle \{\mathcal{I}_i\}_{i \in V}, \{r_{ij}\}_{(i,j) \in E} \rangle$ a model of Σ_d and set $\mathcal{I} := \mathtt{G}(\mathcal{I}_d) = \langle \Delta^{\mathcal{I}}, \cdot^{\mathcal{I}} \rangle$. Then*

$$\#_i(C)^{\mathcal{I}} = C^i, \quad \textit{for every } C \in \widehat{\mathcal{C}}_i,\ i \in V.$$

Proof:

This will follow directly from Lemma 12 once it is shown that $\mathcal{I}_d = \mathtt{F}(\mathtt{G}(\mathcal{I}_d))$. We have, using the full model names to keep notation clear,

- For all $i \in V$, $\Delta^{\mathtt{F}(\mathtt{G}(\mathcal{I}_d))_i} = \top_i^{\mathtt{G}(\mathcal{I}_d)} = \Delta^{(\mathcal{I}_d)_i}$.
- For every $C \in \mathcal{C}_i$, $C^{\mathtt{F}(\mathtt{G}(\mathcal{I}_d))_i} = C^{\mathtt{G}(\mathcal{I}_d)} = C^{(\mathcal{I}_d)_i}$.
- For every $R \in \mathcal{R}_i$, $R^{\mathtt{F}(\mathtt{G}(\mathcal{I}_d))_i} = R^{\mathtt{G}(\mathcal{I}_d)} = R^{(\mathcal{I}_d)_i}$.
- For every $(i,j) \in E$, $r_{ij}^{[\mathtt{F}(\mathtt{G}(\mathcal{I}_d))]} = R_{ij}^{\mathtt{G}(\mathcal{I}_d)} = r_{ij}^{[\mathcal{I}_d]} = r_{ij}$, where the superscripts of r_{ij}'s specify which model they are part of.

Therefore, we do indeed have $\mathcal{I}_d = \mathtt{F}(\mathtt{G}(\mathcal{I}_d))$. Hence, by Lemma 12, $\#_i(C)^{\mathcal{I}} = \#_i(C)^{\mathtt{G}(\mathcal{I}_d)} = C^{\mathtt{F}(\mathtt{G}(\mathcal{I}_d))_i} = C^{(\mathcal{I}_d)_i} = C^i$. ■

The main goal of this section is to show that the converse of Theorem 13 also holds. That is, if an F-$\mathcal{ALCI}$ ontology Σ_d is consistent as witnessed by a specific module T_w, then the corresponding local top $\top_w$ is satisfiable with respect to $\mathfrak{R}(T_w^*)$. More precisely, we have the following

Theorem 18 (Completeness) *Let $\Sigma_d = \{T_i\}_{i \in V}$ be an F-$\mathcal{ALCI}$ ontology. If Σ_d is consistent as witnessed by a module T_w, then $\top_w$ is satisfiable with respect to $\mathfrak{R}(T_w^*)$.*

Proof:

Suppose that Σ_d is consistent as witnessed by T_w. Thus, it has a model $\mathcal{I}_d = \langle \{\mathcal{I}_i\}_{i \in V}, \{r_{ij}\}_{(i,j) \in E} \rangle$, such that $\Delta^w \neq \emptyset$. We proceed to show that $\mathcal{I} := \mathtt{G}(\mathcal{I}_d)$ is a model of $\mathfrak{R}(T_w^*)$, such that $\top_w^{\mathcal{I}} \neq \emptyset$.

We have $\top_w^{\mathcal{I}} = \Delta^w \neq \emptyset$, by the hypothesis.

Clearly, if $C \in \mathcal{C}_i$, then $C^{\mathcal{I}} = C^i \subseteq \Delta^i = \top_i^{\mathcal{I}}$, whence $C \sqsubseteq \top_i$ holds in $\mathcal{I}$.

Next, suppose that $R \in \mathcal{R}_i$ and let $x \in \Delta^{\mathcal{I}} = \bigcup_{i \in V} \Delta^i$. Assume that $y \in \Delta^{\mathcal{I}}$, such that $(x,y) \in R^{\mathcal{I}-}$, i.e., $(y,x) \in R^{\mathcal{I}} = R^i$. Thus, we must have $y \in \Delta^i = \top_i^{\mathcal{I}}$, whence $x \in \{t \in \Delta^{\mathcal{I}} : (\forall y \in \Delta^{\mathcal{I}})((t,y) \in R^{\mathcal{I}} \rightarrow y \in \top_i^{\mathcal{I}})\} = (\forall R^-.\top_i)^{\mathcal{I}}$. This shows that $\top \sqsubseteq \forall R^-.\top_i$ also holds in $\mathcal{I}$. The fact that $\mathcal{I} \models \top \sqsubseteq \forall R.\top_i$ may be shown similarly. Also along the same lines follow the proofs that the two concept inclusion axioms $\top \sqsubseteq \forall R_{ij}^-.\top_i$ and $\top \sqsubseteq \forall R_{ij}.\top_j$ are valid in $\mathcal{I}$.

Finally, suppose that $\#_i(C) \sqsubseteq \#_i(D)$ is in $\mathfrak{R}(\Sigma_d)$. Then $C \sqsubseteq D \in T_i$ and, since $\mathcal{I}_d \models \Sigma_d$, we must have $C^i \subseteq D^i$. Therefore, by Lemma 17, $\#_i(C)^{\mathcal{I}} \subseteq \#_i(D)^{\mathcal{I}}$, which shows that $\mathcal{I} \models \#_i(C) \sqsubseteq \#_i(D)$. Thus, if $\mathcal{I}_d \models T_w^*$, we must have that $\mathtt{G}(\mathcal{I}_d) \models \mathfrak{R}(T_w^*)$. This concludes the proof that, if Σ_d is consistent as witnessed by a package T_w, then $\top_w$ is satisfiable with respect to $\mathfrak{R}(T_w^*)$. ■

As far as exact completeness is concerned, we have

Theorem 19 (Exact Completeness) *Let $\Sigma_d = \{T_i\}_{i \in V}$ be an F-$\mathcal{ALCI}$ ontology. If Σ_d is exactly consistent as witnessed by T_w, then $\top_w$ is satisfiable with respect to $\mathfrak{R}_e(T_w^*)$.*

Proof:

Suppose that Σ_d is exactly consistent as witnessed by T_w. Thus, there exists an exact model $\mathcal{I}_d = \langle \{\mathcal{I}_i\}_{i \in V}, \{r_{ij}\}_{(i,j) \in E} \rangle$ of T_w^*, such that $\Delta^w \neq \emptyset$. We proceed to show that $\mathcal{I} := \mathtt{G}(\mathcal{I}_d)$ is a model of $\mathfrak{R}_e(T_w^*)$, such that $\top_w^{\mathcal{I}} \neq \emptyset$.

By the consistency of Σ_d as witnessed by T_w, we have

- $\top_w^{\mathcal{I}} \neq \emptyset$,
- $\mathcal{I} \models C \sqsubseteq \top_i$, for every $C \in \mathcal{C}_i$,
- $\mathcal{I} \models \top \sqsubseteq \forall R^-.\top_i$ and $\mathcal{I} \models \top \sqsubseteq \forall R.\top_i$, for every $R \in \mathcal{R}_i$,
- $\mathcal{I} \models \top \sqsubseteq \forall R_{ij}^-.\top_i$ and $\mathcal{I} \models \top \sqsubseteq \forall R_{ij}.\top_j$, for every $R_{ij}, (i,j) \in E_w$, and that
- $\mathcal{I} \models \#_i(C) \sqsubseteq \#_i(D)$, for every $C \sqsubseteq D \in T_i$, $i \in V_w$.

Thus, it only suffices to show that, for every importing diagram in T_w^* of the form

$$\begin{tikzcd} & k \arrow[dl] \arrow[dr] & \\ i \arrow[rr] & & j \end{tikzcd}$$

and all $C \in \overline{\mathcal{C}}_i \cap \overline{\mathcal{C}}_j \cap \overline{\mathcal{C}}_k$, $\mathcal{I} \models \exists R_{ij}^-.(\exists R_{ki}^-.\#_k(C)) = \exists R_{kj}^-.\#_k(C)$. We have

$$\begin{aligned}
(\exists R_{ij}^-.(\exists R_{ki}^-.\#_k(C)))^{\mathcal{I}} &= R_{ij}^{\mathcal{I}}(R_{ki}^{\mathcal{I}}(\#_k(C)^{\mathcal{I}})) \quad \text{(by Lemma 10)} \\
&= r_{ij}(r_{ki}(C^k)) \quad \text{(by the definition of } \mathcal{I} \text{ and Lemma 17)} \\
&= r_{kj}(C^k) \quad \text{(since } \mathcal{I}_d \models^e T_w^*\text{, see Lemma 6)} \\
&= R_{kj}^{\mathcal{I}}(\#_k(C)^{\mathcal{I}}) \quad \text{(by the definition of } \mathcal{I} \\
&\qquad\qquad\qquad\qquad \text{and Lemma 17)} \\
&= (\exists R_{kj}^-.\#_k(C))^{\mathcal{I}}. \quad \text{(by Lemma 10)}
\end{aligned}$$

Thus, if $\mathcal{I}_d \models^e T_w^*$, we must have that $\mathtt{G}(\mathcal{I}_d) \models \mathfrak{R}_e(T_w^*)$. This concludes the proof that, if Σ_d is exactly consistent as witnessed by a package T_w, then $\top_w$ is satisfiable with respect to $\mathfrak{R}_e(T_w^*)$. ■

7 Consequences of Soundness and Completeness

By combining Theorems 13 and 18 we get the following

Theorem 20 (Soundness and Completeness) *Suppose that $\Sigma_d = \{T_i\}_{i \in V}$ is an F-$\mathcal{ALCI}$ ontology. Σ_d is consistent as witnessed by a module T_w if and only if $\top_w$ is satisfiable with respect to $\mathfrak{R}(T_w^*)$. Moreover, Σ_d is exactly consistent as witnessed by T_w if and only if $\top_w$ is satisfiable with respect to $\mathfrak{R}_e(T_w^*)$.*

By [30] the concept satisfiability, concept subsumption and consistency problems for the language $\mathcal{ALC}$ are PSPACE-complete. By [33], the same problems for the language $\mathcal{ALCIQ}b$ are in PSPACE. Thus, concept satisfiability, concept subsumption and consistency for the language $\mathcal{ALCI}$ are PSPACE-complete. Since the reductions $\mathfrak{R}$ and $\mathfrak{R}_e$ are obviously doable in polynomial time, we obtain

Theorem 21 *The concept satisfiability, concept subsumption and consistency problems for F-$\mathcal{ALCI}$ are* PSPACE*-complete.*

The following theorem, which is a consequence of Theorem 20, shows that a given subsumption is valid as witnessed by a module T_i of an F-$\mathcal{ALCI}$ ontology T if and only if its translation under $\#_i$ is valid with respect to the reduction $\mathfrak{R}(T_i^*)$. In this case, we say that $\mathfrak{R}$ is a *subsumption-preserving reduction*. Note that this term refers to preservation of subsumptions when passing from a federated ontology to its corresponding single module counterpart and not to a preservation of subsumptions across modules.

Theorem 22 (Subsumption Preservation) *For an F-$\mathcal{ALCI}$ ontology $\Sigma_d = \{T_i\}_{i\in V}$, $T_i^* \models C \sqsubseteq D$ iff $\mathfrak{R}(T_i^*) \models \#_i(C) \sqsubseteq \#_i(D)$.*

Proof:
Suppose, first, that $T_i^* \models C \sqsubseteq D$ and let $\mathcal{I}$ be a model of $\mathfrak{R}(T_i^*)$. Then, by Theorem 13, $\mathtt{F}(\mathcal{I})$ is a model of T_i^*, whence, since $T_i^* \models C \sqsubseteq D$, we get that $\mathtt{F}(\mathcal{I}) \models C \sqsubseteq D$. This implies that $\mathcal{I} \models \#_i(C) \sqsubseteq \#_i(D)$. Therefore $\mathfrak{R}(T_i^*) \models \#_i(C) \sqsubseteq \#_i(D)$.

Conversely, assume that $\mathfrak{R}(T_i^*) \models \#_i(C) \sqsubseteq \#_i(D)$ and let $\mathcal{I}_d \models T_i^*$. Then, by Theorem 18, $\mathtt{G}(\mathcal{I}_d) \models \mathfrak{R}(T_i^*)$, whence, since $\mathfrak{R}(T_i^*) \models \#_i(C) \sqsubseteq \#_i(D)$, we get that $\mathtt{G}(\mathcal{I}_d) \models \#_i(C) \sqsubseteq \#_i(D)$. This implies that $\mathcal{I}_d \models C \sqsubseteq D$. Therefore $T_i^* \models C \sqsubseteq D$. ■

The next consequence of Theorem 20 that we prove concerns the *monotonicity* of federated reasoning *with respect to exact models*. More precisely, we show that, given an F-$\mathcal{ALCI}$ ontology $\Sigma_d = \{T_i\}_{i\in V}$ and an exact model $\mathcal{I}_d$ of Σ_d, a subsumption $C \sqsubseteq D$, with $C, D \in \overline{\mathcal{C}}_i \cap \overline{\mathcal{C}}_j$, $(i,j) \in E$, is valid as witnessed by module T_j provided that it is valid as witnessed by module T_i.

Theorem 23 (Monotonicity) *Let $\Sigma_d = \{T_i\}_{i\in V}$ be an F-$\mathcal{ALCI}$ ontology and $\mathcal{I}_d = \langle \{\mathcal{I}_i\}_{i\in V}, \{r_{ij}\}_{(i,j)\in E}\rangle$ an exact model of Σ_d. Then, for every $(i,j) \in E$ and $C, D \in \overline{\mathcal{C}}_i \cap \overline{\mathcal{C}}_j$, if $C \sqsubseteq_i D$, then $C \sqsubseteq_j D$.*

Proof:
Suppose that $C \sqsubseteq_i D$. Thus, for every model $\mathcal{I}$ of T_i^*, $C^i \subseteq D^i$. Now consider a model $\mathcal{I}$ of T_j^*. Since $(i,j) \in E$, $\mathcal{I}$ is also a model of T_i^*. Therefore, we have that $C^i \subseteq D^i$. Hence, we obtain that $r_{ij}(C^i) \subseteq r_{ij}(D^i)$, whence, by Exactness (see Definition 8), $C^j \subseteq D^j$. This proves that $C \sqsubseteq_j D$. ■

It should be stressed that this theorem has a significant limitation. Monotonicity is asserted only for subsumptions that are actually appearing in two different modules of the ontology under consideration. It cannot be asserted for arbitrary subsumptions that may be added later to the ontology. This is due to the fact that, even if the current model is still a model of the augmented federated ontology, it might not be an exact model. Thus, monotonicity is not being applied to arbitrary concept subsumptions in this case, as was done, for instance, in the case of P-DLs. On the other hand the exactness conditions imposed in the present setting are considerably milder than the ones imposed on the P-DL semantics.

In the special case where $D = \bot$, Theorem 23 yields the following corollary:

Corollary 24 (Preservation of Unsatisfiability) *Let $\Sigma_d = \{T_i\}_{i \in V}$ be an F-$\mathcal{ALCI}$ ontology and $\mathcal{I}_d = \langle \{\mathcal{I}_i\}_{i \in V}, \{r_{ij}\}_{(i,j) \in E} \rangle$ an exact model of Σ_d. Then, for all $(i,j) \in E$ and all $C \in \overline{\mathcal{C}}_i \cap \overline{\mathcal{C}}_j$, if $C \sqsubseteq_i \bot$ then $C \sqsubseteq_j \bot$.*

Thus a concept subsumption $C \sqsubseteq D$, that is unsatisfiable as witnessed by a module T_i, will also be unsatisfiable as witnessed by any other module T_j that imports T_i and shares with T_i the same concepts C, D.

In [5] we listed several desirable properties that modular ontologies may satisfy and imposed various conditions on the interpretations of P-DLs to enforce these desiderata. The list included preservation of unsatisfiability, transitive reusability of knowledge (which is a consequence of monotonicity) and contextualized interpretation of knowledge. In the present work, we chose to consider arbitrary models that do not necessarily satisfy either monotonicity of reasoning or preservation of unsatisfiability. Contextualization of knowledge is satisfied by default. By restricting to exact interpretations, the first two properties in the list reemerge.

Summary

In this paper we have introduced a modular ontology language, contextualized federated description logic F-$\mathcal{ALCI}$, that allows reuse of knowledge from multiple ontologies. An F-$\mathcal{ALCI}$ ontology consists of multiple ontology modules each of which can be viewed as an $\mathcal{ALCI}$ ontology. Concept and role names can be shared by "importing" relations among modules.

The proposed language supports contextualized interpretations, i.e., interpretations from the point of view of a specific module. We have insisted on very loose constraints on image domain relations, i.e., the relations between individuals in different local domains, while still retaining harmonious coordination between the local ontology modules. However, if additional properties are desired, such as the preservation of satisfiability of concept expressions, the monotonicity of inference, or the transitive reusability of knowledge, then more restrictive conditions have to be imposed on the proposed semantics. We have shown how this can be achieved in a case of particular interest.

Ongoing work is aimed at developing a distributed reasoning algorithm for F-$\mathcal{ALCI}$ by extending the results of [1, 3] and [28].

References

1. Jie Bao, Doina Caragea, and Vasant Honavar. A tableau-based federated reasoning algorithm for modular ontologies. In *IEEE/WIC/ACM International Conference on Web Intelligence*, pages 404–410. IEEE Press, 2006.
2. Jie Bao, Giora Slutzki, and Vasant Honavar. A semantic importing approach to knowledge reuse from multiple ontologies. In *AAAI*, pages 1304–1309, 2007.
3. Jie Bao, George Voutsadakis, Giora Slutzki, and Vasant Honavar. Federated reasoning with modular $\mathcal{ALCP}_C$ ontologies. 2008.

4. Jie Bao, George Voutsadakis, Giora Slutzki, and Vasant Honavar. On the decidability of role mappings between modular ontologies. *AAAI*, pages 400–405, 2008.
5. Jie Bao, George Voutsadakis, Giora Slutzki, and Vasant Honavar. Package based description logics. In C. Parent, S. Spaccapietra, and H. Stuckenschmidt, editors, *Ontology Modularization*. Springer-Verlag, In Press.
6. Djamal Benslimane, Catherine Roussey, Christelle Vangenot, and Ahmed Arara. Towards a contextual content of ontologies. In *ISMIS*, pages 339–343, 2003.
7. Tim Berners-Lee, James Hendler, and Ora Lassila. The semantic web. *Scientific American*, 284(5):34–43, May 2001.
8. Elena Paslaru Bontas. Context representation and usage for the semantic web: A state of the art. Technical Report B-04-30, Freie Universität Berlin, http://www.inf.fu-berlin.de/inst/pubs/tr-b-04-30.abstract.html, 2004.
9. Alexander Borgida and Luciano Serafini. Distributed description logics: Directed domain correspondences in federated information sources. In *CoopIS*, pages 36–53, 2002.
10. Paolo Bouquet, Fausto Giunchiglia, Frank van Harmelen, Luciano Serafini, and Heiner Stuckenschmidt. C-OWL: Contextualizing ontologies. In Dieter Fensel, Katia P. Sycara, and John Mylopoulos, editors, *International Semantic Web Conference*, volume 2870 of *Lecture Notes in Computer Science*, pages 164–179. Springer, 2003.
11. Sasa Buvac. Quantificational logic of context. In *AAAI/IAAI, Vol. 1*, pages 600–606, 1996.
12. Sasa Buvac, Vanja Buvac, and Ian A. Mason. Metamathematics of contexts. *Fundam. Inform.*, 23(2/3/4):263–301, 1995.
13. Sasa Buvac and Megumi Kameyama. Introduction: Toward a unified theory of context? *Journal of Logic, Language and Information*, 7(1):1, 1998.
14. Sasa Buvac and Ian A. Mason. Propositional logic of context. In *AAAI*, pages 412–419, 1993.
15. Chiara Ghidini and Fausto Giunchiglia. Local models semantics, or contextual reasoning=locality+compatibility. *Artificial Intelligence*, 127(2):221–259, 2001.
16. Chiara Ghidini and Luciano Serafini. A context-based logic for distributed knowledge representation and reasoning. In *CONTEXT*, pages 159–172, 1999.
17. Chiara Ghidini and Luciano Serafini. Mapping properties of heterogeneous ontologies. In *1st International Workshop on Modular Ontologies (WoMo 2006), co-located with ISWC*, 2006.
18. Silvio Ghilardi, Carsten Lutz, and Frank Wolter. Did i damage my ontology? a case for conservative extensions in description logics. In *KR*, pages 187–197. AAAI Press, 2006.
19. Fausto Giunchiglia and Chiara Ghidini. Local models semantics, or contextual reasoning = locality + compatibility. In *KR*, pages 282–291, 1998.
20. Bernardo Cuenca Grau, Ian Horrocks, Yevgeny Kazakov, and Ulrike Sattler. Modular reuse of ontologies: Theory and practice. *Journal of Artificial Intelligence Research*, 31:273–318, 2008.
21. Bernardo Cuenca Grau, Ian Horrocks, Oliver Kutz, and Ulrike Sattler. Will my ontologies fit together? In *Proc. of the 2006 Description Logic Workshop (DL 2006)*, volume 189. CEUR (http://ceur-ws.org/), 2006.
22. Bernardo Cuenca Grau, Bijan Parsia, and Evren Sirin. Working with multiple ontologies on the semantic web. In *International Semantic Web Conference*, pages 620–634, 2004.
23. Ramanathan V. Guha and John McCarthy. Varieties of contexts. In *CONTEXT*, pages 164–177, 2003.

24. Carsten Lutz, Dirk Walther, and Frank Wolter. Conservative extensions in expressive description logics. In *IJCAI*, pages 453–458, 2007.
25. John McCarthy. Circumscription - a form of non-monotonic reasoning. *Artif. Intell.*, 13(1-2):27–39, 1980.
26. John L. McCarthy. Generality in artificial intelligence. *Commun. ACM*, 30(12):1029–1035, 1987.
27. Drew V. McDermott and Jon Doyle. Non-monotonic logic i. *Artif. Intell.*, 13(1-2):41–72, 1980.
28. Jeff Pan, Luciano Serafini, and Yuting Zhao. Semantic import: An approach for partial ontology reuse. In *1st International Workshop on Modular Ontologies (WoMo 2006), co-located with ISWC*, 2006.
29. Raymond Reiter. A logic for default reasoning. *Artif. Intell.*, 13(1-2):81–132, 1980.
30. Manfred Schmidt-Schauß and Gert Smolka. Attributive concept descriptions with complements. *Artif. Intell.*, 48(1):1–26, 1991.
31. Heiko Stoermer. Introducing Context into Semantic Web Knowledge Bases. In *Proceedings of the CAISE*06 Doctoral Consortium*, June 2006.
32. Heiko Stoermer, Ignazio Palmisano, Domenico Redavid, Luigi Iannone, Paolo Bouquet, and Giovanni Semeraro. Contextualization of a rdf knowledge base in the vikef project. In *ICADL*, pages 101–110, 2006.
33. Stephan Tobies. Complexity results and practical algorithms for logics in knowledge representation. Ph.d. dissertation, Technischen Hochschule Aachen, Aachen, Germany, 2001.
34. Yuting Zhao, Kewen Wang, Rodney W. Topor, Jeff Z. Pan, and Fausto Giunchiglia. Semantic cooperation and knowledge reuse by using autonomous ontologies. In *ISWC/ASWC*, volume 4825 of *Lecture Notes in Computer Science*, pages 666–679. Springer, 2007.