F-$\mathcal{ALCIK}$: Fully Contextualized Federated $\mathcal{ALCI}$ with Epistemic Operators

George Voutsadakis
School of Mathematics and Computer Science
Lake Superior State University
Sault Sainte Marie, MI 49783

November 30, 2011

Abstract

Many semantic web applications require support for knowledge representation and inference over a federation of multiple autonomously developed ontology modules allowing selective partial reuse of knowledge, based on a well-coordinated context-sensitive semantics. Federated $\mathcal{ALCIK}$ or F-$\mathcal{ALCIK}$ is a modular description logic, each of whose modules is roughly an $\mathcal{ALCIK}$ ontology ($\mathcal{ALC}$ with inverse roles and with an epistemic operator). F-$\mathcal{ALCIK}$ supports importing of both concepts and roles across modules and contextualized interpretation of all logical connectives. Moreover, it allows non-monotonic reasoning, formalization of procedural rules as well as various forms of sophisticated query formulation, as does its single-module counterpart $\mathcal{ALCIK}$. To show that this contextualized epistemic federated description logic is decidable and to evaluate the complexity of the corresponding satisfiability problem, a sound and complete reduction to the description logic $\mathcal{ALCIK}$, modeled after a corresponding reduction from F-$\mathcal{ALCI}$ to $\mathcal{ALCI}$, is provided.

1 Introduction

This paper belongs to the tradition of exploring various description logics as the foundation for implementing ontology languages to represent ontologies in the semantic web.

Whereas the actual world-wide web deals mostly with information that is geared towards human understanding and consumption, the vast amounts of information that have recently been accumulating in the web require that they be amenable to processing by machines. The effort to usher this new era of machine interpretable and machine interoperable resources and services in the web has led to the paradigm of the *semantic web* [6]. In the semantic web, information is organized in the form of ontologies and, most often, ontologies are constructed using ontology languages, such as OIL and OWL [15]. Almost all ontology languages have as their theoretical foundation Description Logics (DLs) [9, 10], a family of logic-based knowledge representation languages, that are decidable fragments of first-order logic or decidable extensions of those fragments with various added constructs [1].

Non-monotonic reasoning and procedural rules, which were among many of the desired features of frame-based-systems, cannot be directly expressed in fragments of first-order logic. To accommodate these features in DLs, Donini et al. introduced in [12] an *epistemic operator* on top of the DL $\mathcal{ALC}$ (see [22]). This operator effectively extends $\mathcal{ALC}$ to a fragment of first-order non-monotonic modal logic. It achieves the goal of formalizing procedural rules and, in addition, enables more sophisticated forms of query formulation. Among these are various forms of closed-world reasoning. The knowledge operator in [12] is interpreted as in [20, 17]. This enables the queries to refer both to aspects of the external world, as represented by the knowledge base, and to aspects of what the knowledge base knows about the external world.

Ontologies that refer to one big domain of discourse are ordinarily developed by autonomous independent groups with specialized knowledge of several different, but partially overlapping, subdomains of the entire domain. In each of these independent and autonomously developed modules, the terminology and information is context specific and context sensitive. Moreover, coordination is needed so that concepts and terms may be partially reused by various developers working in interrelated contexts, without having to be explicitly redefined or rewritten. These requirements point to the necessity of introducing into terminological languages two additional features that are not present in basic DLs. One is *modularity*, i.e., the ability to develop independently autonomous but interrelated modules, whose concepts can be easily reused, and the other is *contextualization*, i.e., the property that a statement or a query takes a special meaning depending on the context in which it appears or is posed, respectively. Various modular description logics have appeared in the literature to deal with the first requirement. Among the best known are Distributed Description Logics (DDL) [7], $\mathcal{E}$-Connections [13, 16] and Package-Based Description Logics (P-DLs) [5]. As far as context-sensitivity is concerned, it was recognized early in artificial intelligence by McCarthy and his collaborators (see, e.g., [18, 11]). Significant effort has been subsequently invested in contextualizing ontologies (in the non modular setting) [8, 14]. The recently introduced federated description logic F-$\mathcal{ALCI}$ [23, 24] addresses both the need for modular development of ontologies and the need for contextualization. In fact, it has been the first attempt to contextualize all logical connectives and not just logical negation, as had been the case in previously introduced modular DLs.

In the present work, an attempt is made to combine all three desirable features that were mentioned above, namely,

- expressibility of non-monotonic aspects of reasoning and of procedural rules by incorporating epistemic knowledge operators into the language,
- modularity to allow independent autonomous development of coordinated ontology modules and reuse of knowledge across modules and
- contextualization of both the logical connectives and the epistemic concepts and roles,

in a single modular ontology language. This effort builds on previous work on the fully contextualized federated language F-$\mathcal{ALCI}$ [23], which does not contain epistemic operators. The resulting language F-$\mathcal{ALCIK}$ is, to the best of our knowledge, the first language that combines modularity, contextualization and epistemic operators. When only one module is present, F-$\mathcal{ALCIK}$ reduces to the ordinary DL $\mathcal{ALCIK}$, which is an extension with inverse roles of the language $\mathcal{ALCK}$, that was studied in detail in [12]. We introduce a reduction of a given F-$\mathcal{ALCIK}$ knowledge base to an ordinary $\mathcal{ALCIK}$ ontology, which can be performed in polynomial time in the size of the ontology. This reduction is shown to be sound and complete, in the sense that the federated ontology is consistent from a specific module's point of view if the corresponding top concept is satisfiable in the reduced uni-module ontology and vice-versa. Soundness and completeness, together with known decidability and complexity results for $\mathcal{ALCIK}$ [12] allow us to obtain corresponding results for F-$\mathcal{ALCI}$. More precisely, it is shown that the consistency problem for a F-$\mathcal{ALCIK}$ knowledge base is decidable and that it is ExpTime-complete.

The paper is organized as follows: In Section 2 we present the syntax and the semantics of the fully contextualized, federated epistemic language F-$\mathcal{ALCIK}$. The main features are illustrated using some examples. In Section 3, we present a reduction from the federated language F-$\mathcal{ALCIK}$ to its single module counterpart $\mathcal{ALCIK}$. It serves to demonstrate the decidability of the consistency problem for F-$\mathcal{ALCIK}$, based on the decidability of the corresponding problem in $\mathcal{ALCIK}$. An example serves to illustrate the reduction. Section 4 contains the main results concerning the soundness and completeness of the reduction. At the end of the section, some consequences of these results are also explored. Finally, in the Summary section, we review the main contributions of the paper.

2 The Federated Languages F-$\mathcal{ALCI}$ and F-$\mathcal{ALCIK}$

In this work, we use the prefix "F-", standing for **F**ederated, to denote a contextualized federated language, limited to acyclic importing of concepts, roles and contextualized logical connectives (see [23]). Moreover, following [12], we append at the end the letter $\mathcal{K}$ to denote inclusion of an epistemic operator (or many epistemic operators) in the language.

We present the syntax and the semantics of the language F-$\mathcal{ALCIK}$, which extends the language F-$\mathcal{ALCI}$, that was introduced in [23] (see also [24]) and for which a tableau-based algorithm was presented in [25], with the addition of epistemic operators. One operator $\mathbf{K}_i$ is added for each module i and the intended extension of an epistemic concept $\mathbf{K}_i C$ in module i is supposed to be the set of all individuals in the i-th domain that are in the extension of C in all models of the distributed knowledge base under consideration. The framework assumes a fixed domain semantics, i.e., that the domains of all modules are fixed in all local interpretations, and a fixed image domain relation semantics, i.e., that the image domain relations across the various modules are also fixed in all federated interpretations under consideration.

2.1 The Syntax

Suppose a directed acyclic graph $G = \langle V, E \rangle$, with $V = \{1, 2, \ldots, n\}$, is given. The intuition is that its n nodes correspond to local modules of a modular ontology and its edges correspond to the importing relations between these modules. For technical reasons, we add a loop on each vertex of G.

For every node $i \in V$, the signature of the i-language always includes a set $\mathcal{C}_i$ of i-**concept names** and a set $\mathcal{R}_i$ of i-**role names**. We assume that all sets of names are pairwise disjoint. Out of these, a set of *i-concept expressions* $\widehat{\mathcal{C}}_i$ of F-$\mathcal{ALCI}$ and $\widetilde{\mathcal{C}}_i$ of F-$\mathcal{ALCIK}$ and a set of *i-role expressions* $\widehat{\mathcal{R}}_i$ of F-$\mathcal{ALCI}$ and $\widetilde{\mathcal{R}}_i$ of F-$\mathcal{ALCIK}$ are built.

Recall that the description logic $\mathcal{ALCI}$ allows concept expressions that are constructed recursively from its signature symbols, i.e., its role and concept names, using negation, conjunction, disjunction, value and existential restriction and inverses of role names. Its formulas are subsumptions between concept expressions.

The syntax of the description logic F-$\mathcal{ALCI}$ and that of F-$\mathcal{ALCIK}$ are defined as follows:

Definition 1 (Roles and Concepts) *The set of* i-**roles** *or* i-**role expressions** $\widetilde{\mathcal{R}}_i$ *of F-$\mathcal{ALCIK}$ consists of expressions of the form*

$$R,\ \mathbf{K}_j R,\ \text{where } R \in \mathcal{R}_j \cup \mathcal{R}_j^-, (j,i) \in E,$$

where $\mathcal{R}_j^- = \{R^- : R \in \mathcal{R}_j\}$. *The set of* i-**roles** *or* i-**role expressions** $\widehat{\mathcal{R}}_i$ *of F-$\mathcal{ALCI}$ consists of those i-role expressions in $\widetilde{\mathcal{R}}_i$ that do not use the epistemic operators $\mathbf{K}_j$, $(j,i) \in E$.*

The set of i-**concepts** *or* i-**concept expressions** $\widetilde{\mathcal{C}}_i$ *of F-$\mathcal{ALCIK}$ is defined recursively as follows:*

$$A \in \mathcal{C}_j, \top_j, \bot_j, \neg_j C, C \sqcap_j D, C \sqcup_j D, \exists_j R.C, \forall_j R.C, \mathbf{K}_j C, \tag{1}$$

where $(j,i) \in E$, $C, D \in \widetilde{\mathcal{C}}_i \cap \widetilde{\mathcal{C}}_j$ *and* $R \in \widetilde{\mathcal{R}}_i \cap \widetilde{\mathcal{R}}_j$.
The set of i-**concepts** *or* i-**concept expressions** $\widehat{\mathcal{C}}_i$ *of F-$\mathcal{ALCI}$ consists of those i-concept expressions in $\widetilde{\mathcal{C}}_i$ that do not use the epistemic operators $\mathbf{K}_j$, $(j,i) \in E$ (either in front of role expressions or in front of concept expressions).*

Using the concepts and roles of F-$\mathcal{ALCI}$ or F-$\mathcal{ALCIK}$, we define their formulas, as follows:

Definition 2 (Formulas) *The* i-**formulas** *of F-$\mathcal{ALCI}$ are expressions of the form $C \sqsubseteq D$, with $C, D \in \widehat{\mathcal{C}}_i$, for all $i \in V$. Similarly, the* i-**formulas** *of F-$\mathcal{ALCIK}$ are expressions of the form $C \sqsubseteq D$, with $C, D \in \widetilde{\mathcal{C}}_i$, for all $i \in V$.*

An F-$\mathcal{ALCI}$-**TBox** (F-$\mathcal{ALCIK}$-**TBox**) or, simply, **TBox**, when the relevant language is clear from context, is a collection $T = \{T_i\}_{i \in V}$, where T_i is a finite set of i-formulas of F-$\mathcal{ALCI}$ (F-$\mathcal{ALCIK}$, respectively) for all $i \in V$, called the i-**TBox**. Since, in this paper, we do not consider RBoxes or ABoxes, the terms *TBox*, *ontology* and *knowledge base* (KB) will be used interchangeably.

We use, for every $i \in V$, the notation $\overline{\mathcal{R}}_i$ and $\overline{\mathcal{C}}_i$ to denote the set of i-roles and of i-concepts, respectively, that occur in T_i. These could be in $\widehat{\mathcal{R}}_i$ and $\widehat{\mathcal{C}}_i$, respectively, or $\widetilde{\mathcal{R}}_i$ and $\widetilde{\mathcal{C}}_i$, respectively, depending on the language used in T. $\overline{\mathcal{C}}_i$ is a finite subset of $\widehat{\mathcal{C}}_i$ or $\widetilde{\mathcal{C}}_i$, for every $i \in V$, and, similarly for roles. A role name in $\mathcal{R}_j \cap \overline{\mathcal{R}}_i$ or a concept name in $\mathcal{C}_j \cap \overline{\mathcal{C}}_i$ is said to be **imported from** module j **to** module i.

Example 1 (Syntax): Consider a federated knowledge base with two modules T_1 and T_2 over the directed graph consisting of two nodes 1 and 2 and a directed edge from 1 to 2. Let `Faculty`, `Course` be 1-concept names and `Student` a 2-concept name and, also, `teaches` be a 1-role name and `enrolled` a 2-role name. Assume that module T_1 consists of the formulas

$$\begin{aligned} \top_1 &\sqsubseteq \texttt{Faculty} \sqcup_1 \texttt{Course} \\ \texttt{Faculty} \sqcap_1 \texttt{Course} &\sqsubseteq \bot \\ \exists_1 \texttt{teaches.Course} &\sqsubseteq \texttt{Faculty} \end{aligned}$$

The first two formulas express the fact that the extensions of the 1-concept names are disjoint and cover the domain of discourse of the first module. The last formula expresses the fact that every individual of the local domain that teaches a course must be a faculty. Moreover, let T_2 consist of the formulas

$$\begin{aligned} \top_2 &\sqsubseteq \texttt{Faculty} \sqcup_2 \texttt{Course} \sqcup_2 \texttt{Student} \\ \texttt{Faculty} \sqcap_2 \texttt{Course} &\sqsubseteq \bot \\ \texttt{Faculty} \sqcap_2 \texttt{Student} &\sqsubseteq \bot \\ \texttt{Course} \sqcap_2 \texttt{Student} &\sqsubseteq \bot \\ \exists_2 \texttt{enrolled.Student} &\sqsubseteq \texttt{Course} \end{aligned}$$

Again, the first four formulas assert that the extensions of all concept names, as witnessed by the second module, are pairwise disjoint and cover the domain of discourse of the second module. The last formula, on the other hand, expresses the fact that every individual in the domain that has enrolled a student must be a course. Among the several questions that can be posed against this knowledge base, are

(i) the one concerning the satisfiability of the concept expression

$$\exists_1 \texttt{teaches.Course} \sqcap_1 \neg_1 \mathbf{K}_1 \texttt{Faculty}$$

as witnessed by module T_1 and

(ii) the one concerning the satisfiability of

$$\exists_2 \texttt{teaches.} \exists_2 \texttt{enrolled.Student} \sqcap_2 \neg_2 \texttt{Faculty}$$

as witnessed by T_2.

2.2 The Semantics

In this subsection, we present the semantics for the language F-$\mathcal{ALCIK}$. To do this, we need to revisit the semantics of F-$\mathcal{ALCI}$.

Definition 3 (F-$\mathcal{ALCI}$ Interpretation) *An F-$\mathcal{ALCI}$* **interpretation**

$$\mathcal{I} = \langle \{\mathcal{I}_i\}_{i\in V}, \{r_{ij}\}_{(i,j)\in E}\rangle$$

consists of a family $\mathcal{I}_i = \langle \Delta^i, \cdot^i\rangle, i \in V$, of **local interpretations**, *together with a family of* **image domain relations** *$r_{ij} \subseteq \Delta^i \times \Delta^j, (i,j) \in E$, such that $r_{ii} = \mathrm{id}_{\Delta^i}$, for all $i \in V$.*

Notation: For a binary relation $r \subseteq \Delta^i \times \Delta^j$, $X \subseteq \Delta^i$ and $S \subseteq \Delta^i \times \Delta^i$, we set

$$r(X) := \{y \in \Delta^j : (\exists x \in X)((x,y) \in r)\},$$

$$r(S) := \{(z,w) \in \Delta^j \times \Delta^j : (\exists (x,y) \in S)((x,z),(y,w) \in r)\}.$$

A local interpretation function $\cdot^i$ interprets i-role names and i-concept names, as well as $\bot_i$ and $\top_i$, as follows:

- $C^i \subseteq \Delta^i$, for all $C \in \mathcal{C}_i$,
- $R^i \subseteq \Delta^i \times \Delta^i$, for all $R \in \mathcal{R}_i$,
- $\top_i^i = \Delta^i$, $\bot_i^i = \emptyset$.

The interpretations of imported role names and imported concept names are computed by the following rules:

- $C^i = r_{ji}(C^j)$, for all $C \in \mathcal{C}_j \cap \widehat{\mathcal{C}}_i$,
- $R^i = r_{ji}(R^j)$, for all $R \in \mathcal{R}_j \cap \widehat{\mathcal{R}}_i$,
- $\top_j^i = r_{ji}(\Delta^j)$, $\bot_j^i = \emptyset$.

The recursive features of the local interpretation function $\cdot^i$ are as follows:

- $R^{-i} = R^{i^-}$, for all $R \in \mathcal{R}_i$, where $R^{i^-} = \{(y,x) : (x,y) \in R^i\}$
- $(\neg_j C)^i = r_{ji}(\Delta^j \backslash C^j)$
- $(C \sqcap_j D)^i = r_{ji}(C^j \cap D^j)$
- $(C \sqcup_j D)^i = r_{ji}(C^j \cup D^j)$

- $(\exists_j R.C)^i = r_{ji}(\{x \in \Delta^j : (\exists y)((x,y) \in R^j \text{ and } y \in C^j)\})$
- $(\forall_j R.C)^i = r_{ji}(\{x \in \Delta^j : (\forall y)((x,y) \in R^j \text{ implies } y \in C^j)\})$

In case more than one interpretations are under consideration, the superscript ^{i} will be augmented to $^{\mathcal{I}_i}$ to make explicit the interpretation that is referred to in a specific context.

Definition 4 (F-$\mathcal{ALCIK}$ Interpretation) *An F-$\mathcal{ALCIK}$* **interpretation** *is a pair $(\mathcal{I}, \mathcal{W})$, where $\mathcal{I}$ is a F-$\mathcal{ALCI}$ interpretation and $\mathcal{W}$ is a set of F-$\mathcal{ALCI}$ interpretations, such that, for every $\mathcal{J} \in \mathcal{W}$ and every $i, j \in V$,*

- $\Delta^{\mathcal{J}_i} = \Delta^{\mathcal{I}_i}$, *and*
- $r^{\mathcal{J}}_{ij} = r^{\mathcal{I}}_{ij}$, *if $(i,j) \in E$.*

i.e., local domains corresponding to the same module across all interpretations in $\mathcal{W}$ are assumed to be fixed and the same holds for the image domain relations. We define $\Delta^i := \Delta^{\mathcal{J}_i}$, $i \in V$, and $r_{ij} := r^{\mathcal{I}}_{ij}$, $(i,j) \in E$. Because of the two assumptions displayed above, these definitions are not ambiguous.

The pair $(\mathcal{I}, \mathcal{W})$ gives rise to a local interpretation function $\cdot^{\mathcal{I}_i, \mathcal{W}}$, that interprets i-role names and i-concept names, as well as $\bot_i$ and $\top_i$, as follows:

- $C^{\mathcal{I}_i,\mathcal{W}} = C^{\mathcal{I}_i} \subseteq \Delta^i$, for all $C \in \mathcal{C}_i$,
- $R^{\mathcal{I}_i,\mathcal{W}} = R^{\mathcal{I}_i} \subseteq \Delta^i \times \Delta^i$, for all $R \in \mathcal{R}_i$,
- $\top_i^{\mathcal{I}_i,\mathcal{W}} = \Delta^i$, $\bot_i^{\mathcal{I}_i,\mathcal{W}} = \emptyset$.

The interpretations of imported role names and imported concept names are computed by the following rules:

- $C^{\mathcal{I}_i,\mathcal{W}} = r_{ji}(C^{\mathcal{I}_j,\mathcal{W}})$, for all $C \in \mathcal{C}_j \cap \widetilde{\mathcal{C}}_i$,
- $R^{\mathcal{I}_i,\mathcal{W}} = r_{ji}(R^{\mathcal{I}_j,\mathcal{W}})$, for all $R \in \mathcal{R}_j \cap \widetilde{\mathcal{R}}_i$,
- $\top_j^{\mathcal{I}_i,\mathcal{W}} = r_{ji}(\Delta^j)$, $\bot_j^{\mathcal{I}_i,\mathcal{W}} = \emptyset$.

The recursive features of the local interpretation function $\cdot^{\mathcal{I}_i,\mathcal{W}}$ are as follows:

- $R^{-\mathcal{I}_i,\mathcal{W}} = (R^{\mathcal{I}_i,\mathcal{W}})^-$, for all $R \in \widetilde{\mathcal{R}}_i$
- $(\mathbf{K}_j R)^{\mathcal{I}_i,\mathcal{W}} = r_{ji}(\bigcap_{\mathcal{J} \in \mathcal{W}} R^{\mathcal{J}_j,\mathcal{W}})$, for all $R \in \widetilde{\mathcal{R}}_i$
- $(\neg_j C)^{\mathcal{I}_i,\mathcal{W}} = r_{ji}(\Delta^j \backslash C^{\mathcal{I}_j,\mathcal{W}})$

- $(C \sqcap_j D)^{\mathcal{I}_i,\mathcal{W}} = r_{ji}(C^{\mathcal{I}_j,\mathcal{W}} \cap D^{\mathcal{I}_j,\mathcal{W}})$
- $(C \sqcup_j D)^{\mathcal{I}_i,\mathcal{W}} = r_{ji}(C^{\mathcal{I}_j,\mathcal{W}} \cup D^{\mathcal{I}_j,\mathcal{W}})$
- $(\exists_j R.C)^{\mathcal{I}_i,\mathcal{W}} = r_{ji}(\{x \in \Delta^j : (\exists y)((x,y) \in R^{\mathcal{I}_j,\mathcal{W}} \text{ and } y \in C^{\mathcal{I}_j,\mathcal{W}})\})$
- $(\forall_j R.C)^{\mathcal{I}_i,\mathcal{W}} = r_{ji}(\{x \in \Delta^j : (\forall y)((x,y) \in R^{\mathcal{I}_j,\mathcal{W}} \text{ implies } y \in C^{\mathcal{I}_j,\mathcal{W}})\})$
- $(\mathbf{K}_j C)^{\mathcal{I}_i,\mathcal{W}} = r_{ji}(\bigcap_{\mathcal{J}\in\mathcal{W}} C^{\mathcal{J}_j,\mathcal{W}})$

Note that, for every $R \in \widehat{\mathcal{R}}_i$ and every $C \in \widehat{\mathcal{C}}_i$, we have that $R^{\mathcal{I}_i,\mathcal{W}} = R^{\mathcal{I}_i}$ and $C^{\mathcal{I}_i,\mathcal{W}} = C^{\mathcal{I}_i}$, i.e., this definition restricted to the F-$\mathcal{ALCI}$ roles and concepts coincides with the one given previously for F-$\mathcal{ALCI}$ interpretations, if we disregard the set $\mathcal{W}$.

For all $i \in V$, i-**satisfiability**, denoted by $\models_i$, is defined by $(\mathcal{I},\mathcal{W}) \models_i C \sqsubseteq D$ iff $C^{\mathcal{I}_i,\mathcal{W}} \subseteq D^{\mathcal{I}_i,\mathcal{W}}$. Given a TBox $T = \{T_i\}_{i\in V}$, the interpretation $(\mathcal{I},\mathcal{W})$ is a **model** of T_i, written $(\mathcal{I},\mathcal{W}) \models_i T_i$, iff $(\mathcal{I},\mathcal{W}) \models_i \tau$, for every $\tau \in T_i$. Moreover, $(\mathcal{I},\mathcal{W})$ is a **model** of T, written $(\mathcal{I},\mathcal{W}) \models T$, iff $(\mathcal{I},\mathcal{W}) \models_i T_i$, for every $i \in V$. An **epistemic model** of T is a maximal nonempty set $\mathcal{W}$ of interpretations, such that, for every $\mathcal{I} \in \mathcal{W}$, $(\mathcal{I},\mathcal{W}) \models T$.

Let $w \in V$. Define $G_w = \langle V_w, E_w\rangle$ to be the subgraph of G induced by those vertices in G from which w is reachable and $T_w^* := \{T_i\}_{i\in V_w}$. We say that an F-$\mathcal{ALCIK}$-ontology $T = \{T_i\}_{i\in V}$ is **consistent as witnessed by a module** T_w if T_w^* has an epistemic model $\mathcal{W} = \{\mathcal{I}\}$, with $\mathcal{I} = \langle\{\mathcal{I}_i\}_{i\in V_w}, \{r_{ij}\}_{(i,j)\in E_w}\rangle$, such that $\Delta^w \neq \emptyset$. A concept C is **satisfiable as witnessed by** T_w if there is an epistemic model $\mathcal{W}$ of T_w^*, such that $C^{\mathcal{I}_w,\mathcal{W}} \neq \emptyset$, for some $\mathcal{I} \in \mathcal{W}$. A concept subsumption $C \sqsubseteq D$ **is valid as witnessed by** T_w, denoted by $C \sqsubseteq_w D$, if, for every epistemic model $\mathcal{W}$ of T_w^*, and every $\mathcal{I} \in \mathcal{W}$ $C^{\mathcal{I}_w,\mathcal{W}} \subseteq D^{\mathcal{I}_w,\mathcal{W}}$. An alternative notation for $C \sqsubseteq_w D$ is $T_w^* \models_w C \sqsubseteq D$.

Example 2 (Semantics): We answer the two questions posed against the knowledge base presented in Example 1.

(i) The first one concerns the satisfiability of the concept expression

$$\exists_1\mathtt{teaches.Course} \sqcap_1 \neg_1\mathbf{K}_1\mathtt{Faculty} \tag{2}$$

as witnessed by module T_1. Note, first, that the concept expression

$$\exists_1\mathtt{teaches.Course} \sqcap_1 \neg_1\mathtt{Faculty}$$

is not satisfiable as witnessed by T_1, since

$(\exists_1\mathtt{teaches.Course} \sqcap_1 \neg_1\mathtt{Faculty})^{\mathcal{I}_1,\mathcal{W}}$

$$\begin{aligned}&= (\exists_1\texttt{teaches.Course})^{\mathcal{I}_1,\mathcal{W}} \cap (\neg_1\texttt{Faculty})^{\mathcal{I}_1,\mathcal{W}}\\ &\subseteq \texttt{Faculty}^{\mathcal{I}_1,\mathcal{W}} \cap (\neg_1\texttt{Faculty})^{\mathcal{I}_1,\mathcal{W}}\\ &= \emptyset\end{aligned}$$

However, the concept expression (2) is satisfiable as witnessed by T_1. To see this consider a federated epistemic model $\mathcal{W}$, with $\Delta^1 = \{x, y\}$ and $\Delta^2 = \emptyset$. Let $\mathcal{I} \in \mathcal{W}$ be the model such that $x \in \texttt{Faculty}^{\mathcal{I}_1,\mathcal{W}}$, $y \in \texttt{Course}^{\mathcal{I}_1,\mathcal{W}}$ and $\texttt{teaches}^{\mathcal{I}_1,\mathcal{W}} = \{(x, y)\}$. Moreover, let $\mathcal{J} \in \mathcal{W}$ be the model with $x, y \in \texttt{Course}^{\mathcal{J}_1,\mathcal{W}}$ and $\texttt{Faculty}^{\mathcal{J}_1,\mathcal{W}} = \texttt{teaches}^{\mathcal{J}_1,\mathcal{W}} = \emptyset$. Then, it is not difficult to see that

$$(\exists_1\texttt{teaches.Course} \sqcap_1 \neg_1\mathbf{K}_1\texttt{Faculty})^{\mathcal{I}_1,\mathcal{W}} = \{x\} \neq \emptyset.$$

Thus, the concept expression (2) is satisfiable as witnessed by T_1.

(ii) The second one concerns the satisfiability of

$$\exists_2\texttt{teaches.}\exists_2\texttt{enrolled.Student} \sqcap_2 \neg_2\texttt{Faculty} \tag{3}$$

as witnessed by T_2. Let $(\mathcal{I}, \mathcal{W})$ be an epistemic model of T, such that

$$(\exists_2\texttt{teaches.}\exists_2\texttt{enrolled.Student} \sqcap_2 \neg_2\texttt{Faculty})^{\mathcal{I}_2,\mathcal{W}} \neq \emptyset.$$

Thus, there exists an $x \in \Delta^2$, such that

1. $x \in (\exists_2\texttt{teaches.}\exists_2\texttt{enrolled.Student})^{\mathcal{I}_2,\mathcal{W}}$ and
2. $x \in (\neg_2\texttt{Faculty})^{\mathcal{I}_2,\mathcal{W}} = \Delta^2 \backslash r_{12}(\texttt{Faculty}^{\mathcal{I}_1,\mathcal{W}})$.

Thus, there exist individuals $x', y' \in \Delta^1$, such that $(x', x), (y', y) \in r_{12}$, $(x', y') \in \texttt{teaches}^{\mathcal{I}_1,\mathcal{W}}$ and, also, $y \in (\exists_2\texttt{enrolled.Student})^{\mathcal{I}_2,\mathcal{W}} \subseteq r_{12}(\texttt{Course}^{\mathcal{I}_1,\mathcal{W}})$. But then, by 2 above, $x \notin r_{12}(\texttt{Faculty}^{\mathcal{I}_1,\mathcal{W}})$, whence $x' \notin \texttt{Faculty}^{\mathcal{I}_1,\mathcal{W}}$ and, hence, $y' \notin \texttt{Course}^{\mathcal{I}_1,\mathcal{W}}$. Thus, we get $y' \in \texttt{Faculty}^{\mathcal{I}_1,\mathcal{W}}$. Therefore $y \in r_{12}(\texttt{Faculty}^{\mathcal{I}_1,\mathcal{W}})$, contradicting $y \in r_{12}(\texttt{Course}^{\mathcal{I}_1,\mathcal{W}})$ and $\texttt{Faculty} \sqcap_2 \texttt{Course} \sqsubseteq \bot$. This shows that the concept expression (3) is not satisfiable as witnessed by T_2.

3 A Reduction from F-$\mathcal{ALCIK}$ to $\mathcal{ALCIK}$

In [23] (see also [24]), a **reduction** $\mathfrak{R}$ from an F-$\mathcal{ALCI}$ KB $\Sigma_d = \{T_i\}$ to an $\mathcal{ALCI}$ KB $\Sigma := \mathfrak{R}(\Sigma_d)$ was obtained. Since this reduction is going to be reused in the present work in order to obtain a reduction from an F-$\mathcal{ALCIK}$ KB to an $\mathcal{ALCIK}$ KB, we review it here. This reduction will also allow us to show that the satisfiability problem in F-$\mathcal{ALCIK}$ is decidable, since by [12], the same problem in $\mathcal{ALCIK}$ is decidable, and to pinpoint the complexity of the problem.

Let Σ_d be an F-$\mathcal{ALCI}$ KB. The signature of $\Sigma = \mathfrak{R}(\Sigma_d)$ is the union of the local signatures of the modules together with a global top $\top$, a global bottom $\bot$, local top concepts $\top_i$, for all $i \in V$, and, finally, a collection of new role names $\{R_{ij}\}_{(i,j)\in E}$, i.e.,

$$\mathsf{Sig}(\Sigma) = \bigcup_i (\mathcal{C}_i \cup \mathcal{R}_i) \cup \{\top, \bot\} \cup \{\top_i : 1 \leq i \leq n\} \cup \{R_{ij} : (i,j) \in E\}. \quad (4)$$

Moreover, various axioms derived from the structure of Σ_d are added to Σ.

- For each $C \in \mathcal{C}_i$, $C \sqsubseteq \top_i$ is added to Σ.
- For each $R \in \mathcal{R}_i$, $\top_i$ is stipulated to be the domain and range of R, i.e., $\top \sqsubseteq \forall R^-.\top_i$ and $\top \sqsubseteq \forall R.\top_i$ are added to Σ.
- For each new role name R_{ij}, $\top_i$ is stipulated to be its domain and $\top_j$ to be its range, i.e., $\top \sqsubseteq \forall R_{ij}^-.\top_i$ and $\top \sqsubseteq \forall R_{ij}.\top_j$ are added to Σ.
- For each $C \sqsubseteq D \in T_i$, $\#_i(C) \sqsubseteq \#_i(D)$ is added to Σ, where $\#_i$ is a function from $\widehat{\mathcal{C}_i}$ to the set of $\mathcal{ALCI}$-concepts. The precise definition of $\#_i$ is given below.

The mapping $\#_i(C)$ serves to maintain the compatibility of the concept domains. It is defined by induction on the structure of $C \in \widehat{\mathcal{C}_i}$:

- $\#_i(C) = C$, if $C \in \mathcal{C}_i$;
- $\#_i(C) = \exists R_{ji}^-.\#_j(C)$, if $C \in \mathcal{C}_j \cap \widehat{\mathcal{C}_i}$;
- $\#_i(\neg_j D) = \exists R_{ji}^-.(\neg\, \#_j(D) \sqcap \top_j)$;
- $\#_i(D \boxplus_j E) = \exists R_{ji}^-.(\#_j(D) \boxplus \#_j(E))$, for $\boxplus = \sqcap$ or $\boxplus = \sqcup$;
- $\#_i(\exists_j R.D) = \exists R_{ji}^-.(\exists R_{kj}^-.(\exists R.(\exists R_{kj}.\#_j(D))))$, for $R \in \mathcal{R}_k \cup \mathcal{R}_k^-$;
- $\#_i(\forall_j R.D) = \exists R_{ji}^-.(\forall R_{kj}^-.(\forall R.(\forall R_{kj}.\#_j(D))))$, for $R \in \mathcal{R}_k \cup \mathcal{R}_k^-$.

The reduction $\mathfrak{R}$ was shown in [23] to be sound and complete in the sense that, if the local top concept $\top_w$ in $\mathfrak{R}(\Sigma_d)$, that corresponds to a module T_w in Σ_d, is satisfiable in an $\mathcal{ALCI}$-model of $\mathfrak{R}(\Sigma_d)$, then Σ_d itself is consistent as witnessed by T_w and vice-versa.

Suppose, now, that we start from a F-$\mathcal{ALCIK}$ KB Σ_d. We extend the reduction $\mathfrak{R}$ to cover the contextualized epistemic operators. Thus, we obtain a reduction from the F-$\mathcal{ALCIK}$ KB Σ_d to an $\mathcal{ALCIK}$ KB $\Sigma = \mathfrak{R}(\Sigma_d)$. The signature of Σ is as in Equation (4) and the axioms remain the same. The only definition that needs to be expanded to accommodate the epistemic operators is that of the function $\#_i(C)$. More specifically, the set of clauses above is modified and augmented as follows: We first define two auxiliary operators $\#_i^\exists$ and $\#_i^\forall$ on i-role expressions.

- $\#_i^\exists(R) = \exists R_{ki}^- \exists R \exists R_{ki}$, for all $R \in \mathcal{R}_k \cup \mathcal{R}_k^-$;
- $\#_i^\forall(R) = \forall R_{ki}^- \forall R \forall R_{ki}$, for all $R \in \mathcal{R}_k \cup \mathcal{R}_k^-$;
- $\#_i^\exists(\mathbf{K}_j R) = \exists R_{ji}^- \exists \mathbf{K} R \exists R_{ji}$, for all $R \in \mathcal{R}_j \cup \mathcal{R}_j^-$;
- $\#_i^\forall(\mathbf{K}_j R) = \forall R_{ji}^- \forall \mathbf{K} R \forall R_{ji}$, for all $R \in \mathcal{R}_j \cup \mathcal{R}_j^-$;

Then, using the $\#_i^\exists$ and $\#_i^\forall$-operators, we extend the $\#_i$-operators defined previously for concept expressions in $\widehat{\mathcal{C}}_i$ to concept expressions in $\widetilde{\mathcal{C}}_i$ by induction on the structure of $C \in \widetilde{\mathcal{C}}_i$, as follows:

- $\#_i(C) = C$, if $C \in \mathcal{C}_i$;
- $\#_i(C) = \exists R_{ji}^-.\#_j(C)$, if $C \in \mathcal{C}_j \cap \widehat{\mathcal{C}}_i$;
- $\#_i(\neg_j D) = \exists R_{ji}^-.(\neg\ \#_j(D) \sqcap \top_j)$;
- $\#_i(D \boxplus_j E) = \exists R_{ji}^-.(\#_j(D) \boxplus \#_j(E))$, for $\boxplus = \sqcap$ or $\boxplus = \sqcup$;
- $\#_i(\exists_j R.D) = \exists R_{ji}^-.\#_j^\exists(R).\#_j(D)$, if $R \in \widetilde{\mathcal{R}}_i$;
- $\#_i(\forall_j R.D) = \exists R_{ji}^-.\#_j^\forall(R).\#_j(D)$, if $R \in \widetilde{\mathcal{R}}_i$;
- $\#_i(\mathbf{K}_j D) = \exists R_{ji}^-.\mathbf{K}\#_j(D)$.

Note how the newly introduced notation enables us to handle uniformly in an elegant way the more complex epistemic roles when it comes to the rules dealing with the existential and the universal role quantifications. When only non-epistemic roles are used, the clauses in the definition collapse to the ones used previously for concept expressions in $\widehat{\mathcal{C}}_i$.

Example 3 (Reducing a Federated Knowledge Base): Let us revisit the F-$\mathcal{ALCIK}$ knowledge base T consisting of two modules T_1 and T_2, that was presented in Example 1. We provide below the $\mathcal{ALCIK}$ knowledge base $\mathfrak{R}(T)$, that results from T by applying the reduction that was presented in this section. We use horizontal lines to separate the different categories of translated sentences, as were presented in the definition of $\mathfrak{R}$.

$$
\begin{array}{|l|}
\hline
\texttt{Faculty} \sqsubseteq \top_1 \\
\texttt{Course} \sqsubseteq \top_1 \\
\texttt{Student} \sqsubseteq \top_2 \\
\hline
\top \sqsubseteq \forall \texttt{teaches}^{-}.\top_1 \qquad \top \sqsubseteq \forall \texttt{teaches}.\top_1 \\
\top \sqsubseteq \forall \texttt{enrolled}^{-}.\top_2 \qquad \top \sqsubseteq \forall \texttt{enrolled}.\top_2 \\
\hline
\top \sqsubseteq \forall R_{12}^{-}.\top_1 \qquad \top \sqsubseteq \forall R_{12}.\top_2 \\
\hline
\top_1 \sqsubseteq \texttt{Faculty} \sqcup \texttt{Course} \\
\texttt{Faculty} \sqcap \texttt{Course} \sqsubseteq \bot \\
\exists \texttt{teaches.Course} \sqsubseteq \texttt{Faculty} \\
\top_2 \sqsubseteq \exists R_{12}^{-}.\texttt{Faculty} \sqcup \exists R_{12}^{-}.\texttt{Course} \sqcup \texttt{Student} \\
\exists R_{12}^{-}.\texttt{Faculty} \sqcap \exists R_{12}^{-}.\texttt{Course} \sqsubseteq \bot \\
\exists R_{12}^{-}.\texttt{Faculty} \sqcap \texttt{Student} \sqsubseteq \bot \\
\exists R_{12}^{-}.\texttt{Course} \sqcap \texttt{Student} \sqsubseteq \bot \\
\exists \texttt{enrolled.Student} \sqsubseteq \exists R_{12}^{-}\texttt{Course} \\
\hline
\end{array}
$$

We will show in the next section that the extended reduction $\mathfrak{R}$ is sound and complete in the sense that, given a F-$\mathcal{ALCIK}$ KB $\Sigma_d = \{T_i\}_{i \in V}$, if the local top concept $\top_w$ in $\mathfrak{R}(\Sigma_d)$ is satisfiable in an epistemic model $\mathcal{W} = \{\mathcal{I}\}$, satisfying the property

$$
\begin{cases} \top_i^{\mathcal{I}} = \top_i^{\mathcal{J}}, \text{ for all } i \in V \text{ and all } \mathcal{I}, \mathcal{J} \in \mathcal{W}, \\ R_{ij}^{\mathcal{I}} = R_{ij}^{\mathcal{J}}, \text{ for all } (i,j) \in E \text{ and all } \mathcal{I}, \mathcal{J} \in \mathcal{W}. \end{cases} \tag{5}
$$

then Σ_d itself is consistent as witnessed by T_w and vice-versa.

4 Soundness and Completeness of the Reduction $\mathfrak{R}$

Let us call an epistemic model of $\mathfrak{R}(\Sigma_d)$ satisfying Condition (5) a **coordinated model**. The strategy that we will follow in order to show that the local top concept $\top_w$ in $\mathfrak{R}(\Sigma_d)$ is satisfiable in a coordinated model iff Σ_d is consistent as witnessed by T_w can be summarized as follows: First, given an epistemic coordinated $\mathcal{ALCIK}$ model $\mathcal{W}$ of the KB $\mathfrak{R}(\Sigma_d)$, we will construct an epistemic F-$\mathcal{ALCIK}$ model $\mathtt{F}(\mathcal{W})$ of Σ_d. Then, we will show that if $\top_w$ is satisfiable in the model $\mathcal{W}$, then $\mathtt{F}(\mathcal{W})$ is a model of T^*_w, such that $\Delta^w \neq \emptyset$. Moreover, given an epistemic F-$\mathcal{ALCIK}$ model $\mathcal{W}$ of the KB Σ_d, we will construct an epistemic coordinated $\mathcal{ALCIK}$ model $\mathtt{G}(\mathcal{W})$ of $\Re(\Sigma_d)$ and show that if $\mathcal{W}$ is a model of T^*_w, such that $\Delta^w \neq \emptyset$, then $\top_w$ is satisfiable in the model $\mathtt{G}(\mathcal{W})$.

First, we provide the constructions of $\mathtt{F}(\mathcal{W})$ and $\mathtt{G}(\mathcal{W})$. We urge the reader to keep in mind that, in the first case, the model $\mathcal{W}$ refers to an epistemic model of the $\mathcal{ALCIK}$ KB $\Re(\Sigma_d)$, i.e., $\mathcal{W}$ is a maximal set of $\mathcal{ALCIK}$ interpretations, such that, for all $\mathcal{I} \in \mathcal{W}$, the $\mathcal{ALCIK}$ interpretation $(\mathcal{I}, \mathcal{W})$ is a model of $\Re(\Sigma_d)$, in the sense of [12], satisfying the extra Condition (5). On the other hand, in $\mathtt{G}(\mathcal{W})$, the model $\mathcal{W}$ refers to an epistemic model of the F-$\mathcal{ALCIK}$ KB Σ_d, i.e., $\mathcal{W}$ is a maximal set of F-$\mathcal{ALCIK}$ interpretations, such that, for all $\mathcal{I} \in \mathcal{W}$, the F-$\mathcal{ALCIK}$ interpretation $(\mathcal{I}, \mathcal{W})$ is a model of Σ_d. Hopefully, this mild overloading of notation will not cause any confusion.

Definition 5 *Let $\Sigma_d = \{T_i\}_{i \in V}$ be an F-$\mathcal{ALCIK}$* KB *and $(\mathcal{I}, \mathcal{W})$ an interpretation of the $\mathcal{ALCIK}$* KB *$\Re(\Sigma_d)$, satisfying Condition (5). For each $\mathcal{J} \in \mathcal{W} \cup \{\mathcal{I}\}$, construct an interpretation $\mathtt{F}(\mathcal{J}) = \langle \{\mathcal{J}_i\}_{i \in V}, \{r_{ij}\}_{(i,j) \in E} \rangle$ for Σ_d as follows:*

- $\Delta^{\mathcal{J}_i} = \top_i^{\mathcal{J}}$*, for all $i \in V$;*
- $C^{\mathcal{J}_i} = C^{\mathcal{J}}$*, for every $C \in \mathcal{C}_i$;*
- $R^{\mathcal{J}_i} = R^{\mathcal{J}}$*, for every $R \in \mathcal{R}_i$;*
- $r_{ij} = R_{ij}^{\mathcal{J}}$*, for every $(i, j) \in E$.*

Let $\mathtt{F}(\mathcal{W}) = \{\mathtt{F}(\mathcal{J}) : \mathcal{J} \in \mathcal{W}\}$ and $\mathtt{F}(\mathcal{I}, \mathcal{W}) = (\mathtt{F}(\mathcal{I}), \mathtt{F}(\mathcal{W}))$.

Note how Condition (5) is necessary so that the resulting F-$\mathcal{ALCIK}$ interpretation satisfy the requirements of Definition 4.

Definition 6 *Suppose that Σ_d is an F-$\mathcal{ALCIK}$* KB *and that $(\mathcal{I}, \mathcal{W})$, with $\mathcal{J} = \langle \{\mathcal{J}_i\}_{i \in V}, \{r_{ij}\}_{(i,j) \in E} \rangle$, for all $\mathcal{J} \in \mathcal{W} \cup \{\mathcal{I}\}$, is an interpretation for Σ_d. For every $\mathcal{J} \in \mathcal{W} \cup \{\mathcal{I}\}$, construct an interpretation $\mathtt{G}(\mathcal{J}) = \langle \Delta^{\mathtt{G}(\mathcal{J})}, \cdot^{\mathtt{G}(\mathcal{J})} \rangle$ of $\Re(\Sigma_d)$ as follows:*

- $\Delta^{\mathtt{G}(\mathcal{J})} = \bigcup_{i \in V} \Delta^{\mathcal{J}_i}$*;*
- $\top_i^{\mathtt{G}(\mathcal{J})} = \Delta^{\mathcal{J}_i}$*, for every $i \in V$;*

- $C^{\mathtt{G}(\mathcal{J})} = C^{\mathcal{J}_i}$, *for every* $C \in \mathcal{C}_i$*;*

- $R^{\mathtt{G}(\mathcal{J})} = R^{\mathcal{J}_i}$, *for every* $R \in \mathcal{R}_i$*;*

- $R_{ij}^{\mathtt{G}(\mathcal{J})} = r_{ij}$, *for every* $(i, j) \in E$.

Furtermore, define $\mathtt{G}(\mathcal{W}) = \{\mathtt{G}(\mathcal{J}) : \mathcal{J} \in \mathcal{W}\}$ *and* $\mathtt{G}(\mathcal{I}, \mathcal{W}) = (\mathtt{G}(\mathcal{I}), \mathtt{G}(\mathcal{W}))$.

The following technical lemma that was proven for the case of a F-$\mathcal{ALCI}$ KB Σ_d in [23] (see [24] for proof) is extended here to cover the case of an F-$\mathcal{ALCIK}$ KB Σ_d. It will aid in proving the main soundness and completeness results of this section.

Lemma 7 *Let* Σ_d *be an F-*$\mathcal{ALCIK}$ KB *and* $(\mathcal{I}, \mathcal{W})$ *an interpretation for* $\mathfrak{R}(\Sigma_d)$ *satisfying Condition (5). For every concept expression* $C \in \widetilde{\mathcal{C}}_i$*, such that* $\exists R_{ij}^{-}.C$ *occurs in* $\mathfrak{R}(\Sigma_d)$*, we have* $R_{ij}^{\mathcal{I},\mathcal{W}}(C^{\mathcal{I},\mathcal{W}}) = (\exists R_{ij}^{-}.C)^{\mathcal{I},\mathcal{W}}$.

Proof:
The proof can be carried out almost verbatim as that of Lemma 10 of [24]. We omit the details. ■

To connect the interpretation $(\mathcal{I}, \mathcal{W})$ with its federated counterpart $(\mathtt{F}(\mathcal{I}), \mathtt{F}(\mathcal{W}))$, we need to establish a correspondence between the interpretation of the translation $\#_i(C)$ of a concept $C \in \widetilde{\mathcal{C}}_i$ under $(\mathcal{I}, \mathcal{W})$ and that of the concept C under $(\mathtt{F}(\mathcal{I}), \mathtt{F}(\mathcal{W}))$. This relationship is explored in Lemma 10. For the proof of the lemma, we need to know the exact relationship between the interpretation of a translation of an existential sentence using a complex role expression in a model $(\mathcal{I}, \mathcal{W})$ of $\mathfrak{R}(\Sigma_d)$ and the interpretation of the sentence itself in $\mathtt{F}(\mathcal{I}, \mathcal{W})$. More precisely, we have

Lemma 8 *Let* Σ_d *be an F-*$\mathcal{ALCIK}$ KB, $(\mathcal{I}, \mathcal{W})$ *an interpretation for* $\mathfrak{R}(\Sigma_d)$*, satisfying Condition (5), and* $\exists_i R.D \in \widetilde{\mathcal{C}}_i$*, such that* $\#_i(D)^{\mathcal{I},\mathcal{W}} = D^{\mathtt{F}(\mathcal{I})_i,\mathtt{F}(\mathcal{W})}$. *Then*

$$(\#_i^{\exists}(R).\#_i(D))^{\mathcal{I},\mathcal{W}} = \{x \in \Delta^i : (\exists y \in D^{\mathtt{F}(\mathcal{I})_i,\mathtt{F}(\mathcal{W})})((x, y) \in R^{\mathtt{F}(\mathcal{I})_i,\mathtt{F}(\mathcal{W})})\}.$$

Proof:
Assume, first, that $R \in \mathcal{R}_k \cup \mathcal{R}_k^{-}$. Then, we have

$$(\#_i^{\exists}(R).\#_i(D))^{\mathcal{I},\mathcal{W}}$$

$$
\begin{aligned}
&= (\exists R_{ki}^{-}.\exists R.\exists R_{ki}.\#_i(D))^{\mathcal{I},\mathcal{W}} \\
&= \{x \in \Delta^{\mathcal{I},\mathcal{W}} : (\exists y \in \#_i(D)^{\mathcal{I},\mathcal{W}})((x,y) \in R_{ki}^{\mathcal{I},\mathcal{W}}(R^{\mathcal{I},\mathcal{W}}))\} \\
&= \{x \in \Delta^i : (\exists y \in D^{\mathtt{F}(\mathcal{I})_i,\mathtt{F}(\mathcal{W})})((x,y) \in r_{ki}(R^{\mathtt{F}(\mathcal{I})_k,\mathtt{F}(\mathcal{W})}))\} \\
&= \{x \in \Delta^i : (\exists y \in D^{\mathtt{F}(\mathcal{I})_i,\mathtt{F}(\mathcal{W})})((x,y) \in R^{\mathtt{F}(\mathcal{I})_i,\mathtt{F}(\mathcal{W})})\}.
\end{aligned}
$$

If, on the other hand, $R \in \mathcal{R}_j \cup \mathcal{R}_j^{-}$, we get

$$(\#_i^{\exists}(\mathbf{K}_j R).\#_i(D))^{\mathcal{I},\mathcal{W}}$$

$$
\begin{aligned}
&= (\exists R_{ji}^{-}.\exists \mathbf{K}R.\exists R_{ji}.\#_i(D))^{\mathcal{I},\mathcal{W}} \\
&= \{x \in \Delta^{\mathcal{I},\mathcal{W}} : (\exists y \in \#_i(D)^{\mathcal{I},\mathcal{W}})((x,y) \in R_{ji}^{\mathcal{I},\mathcal{W}}((\mathbf{K}R)^{\mathcal{I},\mathcal{W}}))\} \\
&= \{x \in \Delta^{\mathcal{I},\mathcal{W}} : (\exists y \in \#_i(D)^{\mathcal{I},\mathcal{W}})((x,y) \in R_{ji}^{\mathcal{I},\mathcal{W}}(\textstyle\bigcap_{\mathcal{J}\in\mathcal{W}} R^{\mathcal{J},\mathcal{W}}))\} \\
&= \{x \in \Delta^i : (\exists y \in D^{\mathtt{F}(\mathcal{I})_i,\mathtt{F}(\mathcal{W})})((x,y) \in r_{ji}(\textstyle\bigcap_{\mathcal{J}\in\mathcal{W}} R^{\mathtt{F}(\mathcal{J})_j,\mathtt{F}(\mathcal{W})}))\} \\
&= \{x \in \Delta^i : (\exists y \in D^{\mathtt{F}(\mathcal{I})_i,\mathtt{F}(\mathcal{W})})((x,y) \in (\mathbf{K}_j R)^{\mathtt{F}(\mathcal{I})_i,\mathtt{F}(\mathcal{W})})\}.
\end{aligned}
$$

■

A similar result holds for universal role quantifications. It will also help us in proving Lemma 10, which establishes the main connection between extensions in a coordinated model $(\mathcal{I},\mathcal{W})$ of $\mathfrak{R}(\Sigma_d)$ of translated sentences and corresponding extensions in $\mathtt{F}(\mathcal{I},\mathcal{W})$ of the original sentences.

Lemma 9 *Let Σ_d be an F-$\mathcal{ALCIK}$* KB*, $(\mathcal{I},\mathcal{W})$ be an interpretation for $\mathfrak{R}(\Sigma_d)$, satisfying Condition (5), and $\forall_i R.D \in \widetilde{\mathcal{C}_i}$ such that $\#_i(D)^{\mathcal{I},\mathcal{W}} = D^{\mathtt{F}(\mathcal{I})_i,\mathtt{F}(\mathcal{W})}$. Then*

$$
\begin{aligned}
&(\#_i^{\forall}(R).\#_i(D))^{\mathcal{I},\mathcal{W}} \\
&\quad = \{x \in \Delta^i : (\forall y \in \Delta^i)((x,y) \in R^{\mathtt{F}(\mathcal{I})_i,\mathtt{F}(\mathcal{W})} \textit{ implies } y \in D^{\mathtt{F}(\mathcal{I})_i,\mathtt{F}(\mathcal{W})}))\}.
\end{aligned}
$$

Proof:
Assume, first, that $R \in \mathcal{R}_k \cup \mathcal{R}_k^{-}$. Then, we have

$$(\#_i^{\exists}(R).\#_i(D))^{\mathcal{I},\mathcal{W}}$$

$$
\begin{aligned}
&= (\forall R_{ki}^{-}.\forall R.\forall R_{ki}.\#_i(D))^{\mathcal{I},\mathcal{W}} \\
&= \{x \in \Delta^{\mathcal{I},\mathcal{W}} : (\forall y \in \Delta^{\mathcal{I},\mathcal{W}})((x,y) \in R_{ki}^{\mathcal{I},\mathcal{W}}(R^{\mathcal{I},\mathcal{W}}) \\
&\qquad\qquad \text{implies } y \in \#_i(D)^{\mathcal{I},\mathcal{W}})\} \\
&= \{x \in \Delta^i : (\forall y \in \Delta^i)((x,y) \in r_{ki}(R^{\mathtt{F}(\mathcal{I})_k,\mathtt{F}(\mathcal{W})}) \\
&\qquad\qquad \text{implies } y \in D^{\mathtt{F}(\mathcal{I})_i,\mathtt{F}(\mathcal{W})})\} \\
&= \{x \in \Delta^i : (\forall y \in \Delta^i)((x,y) \in R^{\mathtt{F}(\mathcal{I})_i,\mathtt{F}(\mathcal{W})} \text{ implies } y \in D^{\mathtt{F}(\mathcal{I})_i,\mathtt{F}(\mathcal{W})})\}.
\end{aligned}
$$

If, on the other hand, $R \in \mathcal{R}_j \cup \mathcal{R}_j^{-}$, we get

$$(\#_i^{\forall}(\mathbf{K}_j R).\#_i(D))^{\mathcal{I},\mathcal{W}}$$

$$
\begin{aligned}
&= (\forall R_{ji}^{-}.\forall \mathbf{K}R.\forall R_{ji}.\#_i(D))^{\mathcal{I},\mathcal{W}} \\
&= \{x \in \Delta^{\mathcal{I},\mathcal{W}} : (\forall y \in \Delta^{\mathcal{I},\mathcal{W}})((x,y) \in R_{ji}^{\mathcal{I},\mathcal{W}}((\mathbf{K}R)^{\mathcal{I},\mathcal{W}}) \\
&\qquad\qquad \text{implies } y \in \#_i(D)^{\mathcal{I},\mathcal{W}})\} \\
&= \{x \in \Delta^{\mathcal{I},\mathcal{W}} : (\forall y \in \Delta^{\mathcal{I},\mathcal{W}})((x,y) \in R_{ji}^{\mathcal{I},\mathcal{W}}(\textstyle\bigcap_{\mathcal{J}\in\mathcal{W}} R^{\mathcal{J},\mathcal{W}}) \\
&\qquad\qquad \text{implies } y \in \#_i(D)^{\mathcal{I},\mathcal{W}})\} \\
&= \{x \in \Delta^{i} : (\forall y \in \Delta^{i})((x,y) \in r_{ji}(\textstyle\bigcap_{\mathcal{J}\in\mathcal{W}} R^{\mathtt{F}(\mathcal{J})_j,\mathtt{F}(\mathcal{W})}) \\
&\qquad\qquad \text{implies } y \in D^{\mathtt{F}(\mathcal{I})_i,\mathtt{F}(\mathcal{W})})\} \\
&= \{x \in \Delta^{i} : (\forall y \in \Delta^{i})((x,y) \in (\mathbf{K}_jR)^{\mathtt{F}(\mathcal{I})_i,\mathtt{F}(\mathcal{W})} \\
&\qquad\qquad \text{implies } y \in D^{\mathtt{F}(\mathcal{I})_i,\mathtt{F}(\mathcal{W})})\}.
\end{aligned}
$$

■

Lemma 10 *Suppose that Σ_d is an F-$\mathcal{ALCIK}$ KB, $(\mathcal{I},\mathcal{W})$ an interpretation for $\mathfrak{R}(\Sigma_d)$, satisfying Condition (5), and $\mathtt{F}(\mathcal{I},\mathcal{W})$, with $\mathcal{J} = \langle\{\mathcal{J}_i\}_{i\in V}, \{r_{ij}\}_{(i,j)\in E}\rangle$, for all $\mathcal{J} \in \mathtt{F}(\mathcal{W}) \cup \{\mathtt{F}(\mathcal{I})\}$, such that $\mathcal{J}_i = \langle\Delta^{\mathcal{J}_i}, \cdot^{\mathcal{J}_i}\rangle, i \in V$. Then*

$$\#_i(C)^{\mathcal{I},\mathcal{W}} = C^{\mathtt{F}(\mathcal{I})_i,\mathtt{F}(\mathcal{W})}, \quad \textit{for every } C \in \widetilde{\mathcal{C}}_i,\ i \in V.$$

Proof:

We do this by structural induction on C. All steps of the induction that involve operators other than either the epistemic role or the epistemic concept operators can be carried out exactly as in the proof of Lemma 12 of [24]. Therefore, we will demonstrate in detail only the case where the main (outermost) connective is an existential, universal or epistemic concept operator.

Suppose that $C = \exists_j R.D$, with $R \in \widetilde{\mathcal{R}}_i$. Then

$$
\begin{aligned}
\#_i(\exists_j R.D)^{\mathcal{I},\mathcal{W}} &= (\exists R_{ji}^{-}.\#_j^{\exists}R.\#_j(D))^{\mathcal{I},\mathcal{W}} \\
&\qquad \text{(by the definition of } \#_i(\exists_j R.D)) \\
&= R_{ji}^{\mathcal{I},\mathcal{W}}((\#_j^{\exists}R.\#_j(D))^{\mathcal{I},\mathcal{W}}) \quad \text{(by Lemma 7)} \\
&= r_{ji}(\{x \in \Delta^{j} : (\exists y \in D^{\mathtt{F}(\mathcal{I})_j,\mathtt{F}(\mathcal{W})})((x,y) \in \\
&\qquad\qquad R^{\mathtt{F}(\mathcal{I})_j\mathtt{F}(\mathcal{W})})\}) \\
&\qquad \text{(by Lemma 8 and the induction hypothesis)} \\
&= r_{ji}((\exists_j R.D)^{\mathtt{F}(\mathcal{I})_j,\mathtt{F}(\mathcal{W})}) \\
&= (\exists_j R.D)^{\mathtt{F}(\mathcal{I})_i,\mathtt{F}(\mathcal{W})}.
\end{aligned}
$$

The case of a universal role quantification may be handled similarly. The only difference is that Lemma 9 should be used in place of Lemma 8.

Suppose, finally, that $C = \mathbf{K}_j D$. Then

$$\begin{array}{rcl}
\#_i(\mathbf{K}_j D)^{\mathcal{I},\mathcal{W}} & = & (\exists R_{ji}^-.\mathbf{K}\#_j(D))^{\mathcal{I},\mathcal{W}} \quad \text{(by the definition of } \#_i(\mathbf{K}_j D)) \\
& = & R_{ji}^{\mathcal{I},\mathcal{W}}((\mathbf{K}\#_j(D))^{\mathcal{I},\mathcal{W}}) \quad \text{(by Lemma 7)} \\
& = & R_{ji}^{\mathcal{I},\mathcal{W}}(\bigcap_{\mathcal{J}\in\mathcal{W}} \#_j(D)^{\mathcal{J},\mathcal{W}}) \quad \text{(by the definition of } \cdot^{\mathcal{I},\mathcal{W}}) \\
& = & r_{ji}(\bigcap_{\mathcal{J}\in\mathcal{W}} D^{\mathtt{F}(\mathcal{J})_i,\mathtt{F}(\mathcal{W})}) \quad \text{(by the definition of } \mathtt{F}(\mathcal{I},\mathcal{W}) \\
& & \qquad \text{and the induction hypothesis)} \\
& = & (\mathbf{K}_j D)^{\mathtt{F}(\mathcal{I})_i,\mathtt{F}(\mathcal{W})}. \text{ (by the defintiion of } (\mathbf{K}_j D)^{\mathtt{F}(\mathcal{I})_i,\mathtt{F}(\mathcal{W})})
\end{array}$$

■

The following is our main soundness theorem for the reduction $\mathfrak{R}$.

Theorem 11 (Soundness) *Let Σ_d be an F-$\mathcal{ALCIK}$ KB, and T_w a module of Σ_d. If $\top_w$ is satisfiable in a coordinated model of $\mathfrak{R}(T_w^*)$, then Σ_d is consistent as witnessed by T_w.*

Proof:

Suppose that $\top_w$ is satisfiable in a coordinated model of $\mathfrak{R}(T_w^*)$. Assume $(\mathcal{I},\mathcal{W})$, with $\mathcal{I} = \langle\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}}\rangle$, such that $\top_w^{\mathcal{I}} \neq \emptyset$ is a coordinated model of $\mathfrak{R}(T_w^*)$. Our goal is to show that $(\mathtt{F}(\mathcal{I}),\mathtt{F}(\mathcal{W}))$ is an epistemic model of T_w^*, such that $\Delta^w := \Delta^{\mathtt{F}(\mathcal{I})_w,\mathtt{F}(\mathcal{W})} \neq \emptyset$.

Clearly, we have $\Delta^w = \top_w^{\mathcal{I},\mathcal{W}} \neq \emptyset$, by the hypothesis. So it suffices to show that $(\mathtt{F}(\mathcal{I}),\mathtt{F}(\mathcal{W}))$ is a model of the federated ontology T_w^*, i.e., that it satisfies $(\mathtt{F}(\mathcal{I}),\mathtt{F}(\mathcal{W})) \models_i T_i$, for every $i \in V_w$. Suppose that $C \sqsubseteq D \in T_i$. By the construction of $\mathfrak{R}(T_w^*)$ and the fact that $(\mathcal{I},\mathcal{W}) \models \mathfrak{R}(T_w^*)$, we must have $\#_i(C)^{\mathcal{I},\mathcal{W}} \subseteq \#_i(D)^{\mathcal{I},\mathcal{W}}$, whence, by Lemma 10, we obtain that $C^{\mathtt{F}(\mathcal{I})_i,\mathtt{F}(\mathcal{W})} \subseteq D^{\mathtt{F}(\mathcal{I})_i,\mathtt{F}(\mathcal{W})}$, showing that $(\mathtt{F}(\mathcal{I}),\mathtt{F}(\mathcal{W})) \models T_w^*$. ■

To connect the federated interpretation $(\mathcal{I},\mathcal{W})$ with its single module-counterpart $\mathtt{G}(\mathcal{I},\mathcal{W})$, we need to establish a correspondence between the interpretation of the translation $\#_i(C)$ of a concept $C \in \widetilde{\mathcal{C}_i}$ under $\mathtt{F}(\mathcal{I},\mathcal{W})$ and that of the concept C under $(\mathcal{I},\mathcal{W})$.

Lemma 12 *Let Σ_d be an F-$\mathcal{ALCIK}$ KB and $(\mathcal{I},\mathcal{W})$, with $\mathcal{W} = \{\mathcal{J}\}$, $\mathcal{J} = \langle\{\mathcal{J}_i\}_{i\in V}, \{r_{ij}\}_{(i,j)\in E}\rangle$, an epistemic model of Σ_d. Then*

$$\#_i(C)^{\mathtt{G}(\mathcal{I}),\mathtt{G}(\mathcal{W})} = C^{\mathcal{I}_i,\mathcal{W}}, \text{ for every } C \in \widetilde{\mathcal{C}_i},\ i \in V.$$

Proof:

As in the proof of Lemma 17 of [24], one may show that, for every $\mathcal{J} \in \mathcal{W}$, we have that $\mathcal{J} = \mathtt{F}(\mathtt{G}(\mathcal{J}))$. Therefore, by Lemma 10, $\#_i(C)^{\mathtt{G}(\mathcal{I}),\mathtt{G}(\mathcal{W})} = C^{\mathtt{F}(\mathtt{G}(\mathcal{I}))_i,\mathtt{F}(\mathtt{G}(\mathcal{W}))} = C^{\mathcal{I}_i,\mathcal{W}}$. ■

Having at hand Lemma 12, we can now show that the converse of Theorem 11 also holds. That is, if an F-$\mathcal{ALCIK}$ ontology Σ_d is consistent as witnessed by a specific module T_w, then the corresponding local top $\top_w$ is satisfiable in a coordinated model of $\mathfrak{R}(T_w^*)$. More precisely, we have the following

Theorem 13 (Completeness) *Let $\Sigma_d = \{T_i\}_{i\in V}$ be an F-$\mathcal{ALCIK}$ ontology. If Σ_d is consistent as witnessed by a module T_w, then $\top_w$ is satisfiable in a coordinated model of $\mathfrak{R}(T_w^*)$.*

Proof:

Suppose that Σ_d is consistent as witnessed by T_w. Thus, it has an epistemic model $(\mathcal{I}, \mathcal{W})$, with $\mathcal{J} = \langle\{\mathcal{J}_i\}_{i\in V}, \{r_{ij}\}_{(i,j)\in E}\rangle$, $\mathcal{J} \in \mathcal{W} \cup \{\mathcal{I}\}$, satisfying $\Delta^{\mathcal{I}_w} \neq \emptyset$. We show that $(\mathtt{G}(\mathcal{I}), \mathtt{G}(\mathcal{W}))$ is a model of $\mathfrak{R}(T_w^*)$, such that $\top_w^{\mathtt{G}(\mathcal{I})} \neq \emptyset$.

We have $\top_w^{\mathtt{G}(\mathcal{I}),\mathtt{G}(\mathcal{W})} = \Delta^{\mathcal{I}_w,\mathcal{W}} \neq \emptyset$, by the hypothesis.

Clearly, if $C \in \mathcal{C}_i$, then $C^{\mathtt{G}(\mathcal{I}),\mathtt{G}(\mathcal{W})} = C^{\mathtt{G}(\mathcal{I})} = C^{\mathcal{I}_i,\mathcal{W}} \subseteq \Delta^i = \top_i^{\mathtt{G}(\mathcal{I}),\mathtt{G}(\mathcal{W})}$, whence $C \sqsubseteq \top_i$ holds in $(\mathtt{G}(\mathcal{I}), \mathtt{G}(\mathcal{W}))$.

Next, suppose that $R \in \mathcal{R}_i$ and let $x \in \Delta^{\mathtt{G}(\mathcal{I}),\mathtt{G}(\mathcal{W})} = \Delta^{\mathtt{G}(\mathcal{I})} = \bigcup_{i\in V} \Delta^i$. Assume that $y \in \Delta^{\mathtt{G}(\mathcal{I})}$, such that $(x, y) \in (R^{\mathtt{G}(\mathcal{I}),\mathtt{G}(\mathcal{W})})^- = (R^{\mathtt{G}(\mathcal{I})})^-$, i.e., $(y, x) \in R^{\mathtt{G}(\mathcal{I})} = R^{\mathcal{I}_i,\mathcal{W}}$. Thus, we must have $y \in \Delta^i = \top_i^{\mathtt{G}(\mathcal{I}),\mathtt{G}(\mathcal{W})}$, whence $x \in \{t \in \Delta^{\mathtt{G}(\mathcal{I}),\mathtt{G}(\mathcal{W})} : (\forall y \in \Delta^{\mathtt{G}(\mathcal{I}),\mathtt{G}(\mathcal{W})})((t, y) \in R^{\mathtt{G}(\mathcal{I}),\mathtt{G}(\mathcal{W})} \to y \in \top_i^{\mathtt{G}(\mathcal{I}),\mathtt{G}(\mathcal{W})})\}$ $= (\forall R^-.\top_i)^{\mathtt{G}(\mathcal{I}),\mathtt{G}(\mathcal{W})}$. This shows that $\top \sqsubseteq \forall R^-.\top_i$ also holds in $(\mathtt{G}(\mathcal{I}), \mathtt{G}(\mathcal{W}))$. The fact that $(\mathtt{G}(\mathcal{I}), \mathtt{G}(\mathcal{W})) \models \top \sqsubseteq \forall R.\top_i$ may be shown similarly. Also along the same lines follow the proofs that the two concept inclusion axioms $\top \sqsubseteq \forall R_{ij}^-.\top_i$ and $\top \sqsubseteq \forall R_{ij}.\top_j$ are valid in $(\mathtt{G}(\mathcal{I}), \mathtt{G}(\mathcal{W}))$.

Finally, suppose that $\#_i(C) \sqsubseteq \#_i(D)$ is in $\mathfrak{R}(\Sigma_d)$. Then $C \sqsubseteq D \in T_i$ and, since $(\mathcal{I}, \mathcal{W}) \models \Sigma_d$, we must have $C^{\mathcal{I}_i,\mathcal{W}} \subseteq D^{\mathcal{I}_i,\mathcal{W}}$. Therefore, by Lemma 12, $\#_i(C)^{\mathtt{G}(\mathcal{I}),\mathtt{G}(\mathcal{W})} \subseteq \#_i(D)^{\mathtt{G}(\mathcal{I}),\mathtt{G}(\mathcal{W})}$, whence $(\mathtt{G}(\mathcal{I}), \mathtt{G}(\mathcal{W})) \models \#_i(C) \sqsubseteq \#_i(D)$. Thus, if $(\mathcal{I}, \mathcal{W}) \models T_w^*$, we must have that $(\mathtt{G}(\mathcal{I}), \mathtt{G}(\mathcal{W})) \models \mathfrak{R}(T_w^*)$. This concludes the proof that, if Σ_d is consistent as witnessed by a package T_w, then $\top_w$ is satisfiable with respect to $\mathfrak{R}(T_w^*)$. (Note that $(\mathtt{G}(\mathcal{I}), \mathtt{G}(\mathcal{W}))$ is indeed a coordinated model due to Definition 4.) ■

Consequences of Soundness and Completeness

By combining Theorems 11 and 13 we get the following

Theorem 14 (Soundness and Completeness) *Let $\Sigma_d = \{T_i\}_{i\in V}$ be an F-$\mathcal{ALCIK}$ ontology. Σ_d is consistent as witnessed by a module T_w if and only if $\top_w$ is satisfiable in a coordinated model of $\mathfrak{R}(T_w^*)$.*

It is well known that concept satisfiability in $\mathcal{ALCI}$ with respect to arbitrary TBoxes is ExpTime-hard [21, 2]. Thus concept satisfiability in $\mathcal{ALCIK}$ is also ExpTime-hard. By the results presented in [12], it can be shown that concept satisfiability in $\mathcal{ALCIK}$ is in ExpTime. Therefore, since the reduction $\mathfrak{R}$ can be performed in polynomial time, we obtain

Theorem 15 *Deciding TBox consistency in F-$\mathcal{ALCIK}$ is* ExpTime*-complete.*

The following theorem, which is a consequence of Theorem 14, shows that a given subsumption is valid as witnessed by a module T_i of an F-$\mathcal{ALCIK}$ ontology T if and only if its translation under $\#_i$ is valid in any coordinated model of $\mathfrak{R}(T_i^*)$.

Theorem 16 (Subsumption Preservation) *Let $\Sigma_d = \{T_i\}_{i \in V}$ be an F-$\mathcal{ALCIK}$ ontology. Then $T_i^* \models_i C \sqsubseteq D$ iff $\#_i(C) \sqsubseteq \#_i(D)$ is satisfied in every coordinated model of $\mathfrak{R}(T_i^*)$.*

Proof:

Suppose, first, that $T_i^* \models_i C \sqsubseteq D$ and let $(\mathcal{I}, \mathcal{W})$ be a coordinated epistemic model of $\mathfrak{R}(T_i^*)$. Then, by Theorem 11, $(\mathtt{F}(\mathcal{I}), \mathtt{F}(\mathcal{W}))$ is a model of T_i^*, whence, since $T_i^* \models_i C \sqsubseteq D$, we get that $(\mathtt{F}(\mathcal{I}), \mathtt{F}(\mathcal{W}) \models_i C \sqsubseteq D$. This implies that $(\mathcal{I}, \mathcal{W}) \models \#_i(C) \sqsubseteq \#_i(D)$.

Conversely, suppose that every coordinated model of $\mathfrak{R}(T_i^*)$ satisfies $\#_i(C) \sqsubseteq \#_i(D)$ and let $(\mathcal{I}, \mathcal{W}) \models T_i^*$. Then, by Theorem 13, $(\mathtt{G}(\mathcal{I}), \mathtt{G}(\mathcal{W}))$ is a coordinated model of $\mathfrak{R}(T_i^*)$. Thus, we get that $(\mathtt{G}(\mathcal{I}), \mathtt{G}(\mathcal{W})) \models \#_i(C) \sqsubseteq \#_i(D)$. This implies that $(\mathcal{I}, \mathcal{W}) \models_i C \sqsubseteq D$. Therefore $T_i^* \models_i C \sqsubseteq D$. ■

Summary

In this paper we have introduced a modular ontology language, contextualized federated description logic F-$\mathcal{ALCIK}$, that allows reuse of knowledge from multiple ontologies. An F-$\mathcal{ALCIK}$ ontology consists of multiple ontology modules each of which can be viewed as an $\mathcal{ALCIK}$ ontology and has several important features that, to the best of our knowledge, appear together in a single ontology language for the first time. Namely,

- it is modular, i.e., provides a semantics for independently developed, autonomous but interrelated modules, that can share parts of their vocabulary without the need to redefine their meanings;

- it is fully contextualized, since all its logical connectives assume a different meaning when used in different modules; a module using a "foreign" connective is calling for its extension to be computed in another local domain and then imported via inter-domain relations;

- it provides support for the use of contextualized epistemic operators in each module; whereas the epistemic role operators can be applied only in front of roles defined in their own module, the epistemic concept operators can be applied in front of arbitrary concept expressions, as long as the necessary importing relations are fulfilled.

A case has been made in [12] that the knowledge operators allow non-monotonic reasoning, some forms of closed-world reasoning, formulation of procedural rules as well as various forms of more sophisticated queries. Since our language, when restricted to a single module, has the same semantics as that in [12], these nice properties are inherited in the present context.

F-$\mathcal{ALCIK}$ supports contextualized interpretations, i.e., interpretations from the point of view of a specific module. We have provided a reduction from an F-$\mathcal{ALCIK}$ ontology to an $\mathcal{ALCIK}$ ontology that allowed us to use the results of [12] to show that the consistency problem for a F-$\mathcal{ALCIK}$ ontology is ExpTime-complete.

It is an interesting problem to develop a distributed reasoning algorithm for F-$\mathcal{ALCIK}$ by extending the algorithm for F-$\mathcal{ALCI}$, that was presented in [25]. This algorithm was based, in turn, in similar work of Bao et al. [3, 4] and Pan et al. [19].

References

[1] F. Baader and W. Nutt. Basic description logics. In Franz Baader, Diego Calvanese, and Deborah McGuinness et.al., editors, *The Description Logic Handbook: Theory, Implementation, and Applications*, pages 43–95. Cambridge University Press, 2003.

[2] Franz Baader, Diego Calvanese, Deborah L. McGuinness, Daniele Nardi, and Peter F. Patel-Schneider, editors. *The Description Logic Handbook: Theory, Implementation, and Applications.* Cambridge University Press, 2003.

[3] Jie Bao, Doina Caragea, and Vasant Honavar. A tableau-based federated reasoning algorithm for modular ontologies. In *IEEE/WIC/ACM*

International Conference on Web Intelligence, pages 404–410. IEEE Press, 2006.

[4] Jie Bao, George Voutsadakis, Giora Slutzki, and Vasant Honavar. Distributed reasoning with modular $\mathcal{ALCP}_C$ ontologies. 2008.

[5] Jie Bao, George Voutsadakis, Giora Slutzki, and Vasant Honavar. *Ontology Modularization*, chapter Package based Description Logics. Berlin: Springer (In Press), 2008.

[6] T Berners-Lee, J Hendler, and Ora Lassila. The semantic web. *Scientific American*, 284(5):34–43, May 2001.

[7] Alexander Borgida and Luciano Serafini. Distributed description logics: Assimilating information from peer sources. *Journal of Data Semantics*, 1:153–184, 2003.

[8] Paolo Bouquet, Fausto Giunchiglia, Frank van Harmelen, Luciano Serafini, and Heiner Stuckenschmidt. C-OWL: Contextualizing ontologies. In Dieter Fensel, Katia P. Sycara, and John Mylopoulos, editors, *International Semantic Web Conference*, volume 2870 of *Lecture Notes in Computer Science*, pages 164–179. Springer, 2003.

[9] Ronald J. Brachman, Hector J. Levesque, and Richard Fikes. Krypton: Integrating terminology and assertion. In *AAAI*, pages 31–35, 1983.

[10] Ronald J. Brachman and James G. Schmolze. An overview of the kl-one knowledge representation system. *Cognitive Science*, 9(2):171–216, 1985.

[11] Sasa Buvac, Vanja Buvac, and Ian A. Mason. Metamathematics of contexts. *Fundam. Inform.*, 23(2/3/4):263–301, 1995.

[12] Francesco M. Donini, Maurizio Lenzerini, Daniele Nardi, Werner Nutt, and Andrea Schaerf. An epistemic operator for description logics. *Artificial Intelligence*, 100(1-2):225–274, 1998.

[13] Bernardo Cuenca Grau and Bijan Parsia. From shoq(d) toward e-connections. In *Description Logics*, 2004.

[14] Ramanathan V. Guha, Rob McCool, and Richard Fikes. Contexts for the semantic web. In Sheila A. McIlraith, Dimitris Plexousakis, and Frank van Harmelen, editors, *International Semantic Web Conference*, volume 3298 of *Lecture Notes in Computer Science*, pages 32–46. Springer, 2004.

[15] Ian Horrocks, Peter F. Patel-Schneider, and Frank van Harmelen. From SHIQ and RDF to OWL: the making of a web ontology language. *J. Web Sem.*, 1(1):7–26, 2003.

[16] Oliver Kutz, Carsten Lutz, Frank Wolter, and Michael Zakharyaschev. E-connections of abstract description systems. *Artif. Intell.*, 156(1):1–73, 2004.

[17] Vladimir Lifschitz. Minimal belief and negation as failure. *Artif. Intell.*, 70(1-2):53–72, 1994.

[18] John McCarthy. Notes on formalizing context. In *IJCAI*, pages 555–562, 1993.

[19] Jeff Pan, Luciano Serafini, and Yuting Zhao. Semantic import: An approach for partial ontology reuse. In *1st International Workshop on Modular Ontologies (WoMo 2006), co-located with ISWC*, 2006.

[20] Raymond Reiter. What should a database know? *J. Log. Program.*, 14(1&2):127–153, 1992.

[21] Klaus Schild. A correspondence theory for terminological logics: Preliminary report. In *In Proc. of IJCAI-91*, pages 466–471, 1991.

[22] Manfred Schmidt-Schauß and Gert Smolka. Attributive concept descriptions with complements. *Artif. Intell.*, 48(1):1–26, 1991.

[23] George Voutsadakis, Jie Bao, Giora Slutzki, and Vasant Honavar. Federated $\mathcal{ALCI}$: Preliminary report. *WI* 2008*, To appear*, 2008.

[24] George Voutsadakis, Jie Bao, Giora Slutzki, and Vasant Honavar. F-$\mathcal{ALCI}$: A fully contextualized, federated logic for the semantic web. *Preprint*, 2009.

[25] George Voutsadakis, Giora Slutzki, Jie Bao, and Vasant Honavar. A distributed tableau algorithm for F-$\mathcal{ALCI}$. *Preprint*, 2009.